AAS 11-151

EPHEMERIS REQUIREMENTS FOR SPACE SITUATIONAL AWARENESS

Daniel L. Oltrogge*, T.S. Kelso*, and John H. Seago†

Increasing international cooperation in the areas of orbital collision avoidance and electro-magnetic interference mitigation has driven the exchange of satellite ephemerides to support Space Situational Awareness, threat detection, and avoidance. To be useful, ephemerides must conform to certain specifications in order to ensure that the intended precision of these analyses is achieved. This paper examines the accuracy of various interpolation methods as a function of ephemeris step size and ephemeris numerical precision for a variety of orbital regimes.

INTRODUCTION

As orbital operations and the background space debris population have increased, orbital Collision Avoidance (CA) and Radio Frequency Interference (RFI) mitigation have become increasingly important to space operators. This has successfully led to a number of standards and practices governing the exchange of orbital information among operators, most notably the joint effort of CCSDS/ISO collaborative effort to create CCSDS 502.0-B-2 "Orbit Data Messages."[1] Additionally, commercial operators are jointly addressing the collision and RFI threats by notifying each other of RFI events and by performing predictive threat analyses.

Spacecraft operators typically have the best positional and attitudinal histories and predictions for their satellites. Such operators tend to have very detailed models for solar-radiation pressure, know their mass and drag-relevant cross-sectional areas, know when their maneuvers are to occur, and can tailor their gravity and perturbations modeling to the specific orbital environment their satellite occupies. Optical and/or radar surveillance are necessary to maintain situational awareness when operator-provided ephemerides are unavailable; however, an extrapolation of historical observations is unable to predict future spacecraft maneuvers.

Owner-supplied ephemerides are snapshots in time of where spacecraft are predicted to be. The accuracy of such predictions depends upon a complex combination of factors, including: operator observation precision and accuracy, operator orbit-determination method precision and accuracy, operator integration method, interpolation and output precision, operator orbit perturbations fidelity, operator maneuver calibration and performance, operator timing references and Earth orientation parameters, recipient timing references and Earth orientation parameters, recipi-

* Senior Research Astrodynamicist, AGI's Center for Space Standards and Innovation, 7150 Campus Drive, Suite 260, Colorado Springs, CO 80920.
† Astrodynamics Engineer, Analytical Graphics Inc., 220 Valley Creek Boulevard, Exton, PA 19341.

ent instantiation and rectification of operator ephemeris reference frame, and recipient ephemeris interpolation methods, accuracy and numerical precision.

Note that the recipient has only three aspects in their control to ensure they are getting the best positional accuracy out of an operator's ephemeris. The first two are addressed through careful consistency checks with each operator to ensure that timing and reference frames are consistently defined. The third addresses the interpolation method, the ephemeris step size, the introduction of maneuvers into the ephemeris, and the numerical precision required to produce the desired accuracy for such analyses. The impact of each of these aspects will be examined for a variety of orbital regimes to help provide the analyst and space operators with guidance for ephemeris creation and usage.

BENEFITS OF TRUSTWORTHY SATELLITE POSITIONAL EPHEMERIS

The operator's ephemeris is used in both CA and RFI geolocation analyses. One question is, "How sensitive are the outcomes of these analyses to errors in satellite position?" For CA, it is straightforward to show that the relative position error could range from zero up to the sum of the position errors for both satellites. It may be less apparent that the computed *time of closest approach* (TCA) will also be affected, since the relative motion geometry and resultant close approach have likely shifted in time.

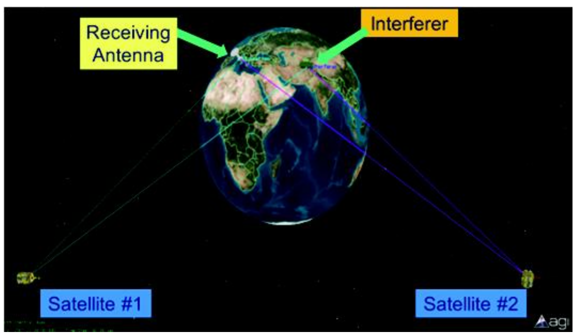


Figure 1. RFI geolocation geometry.

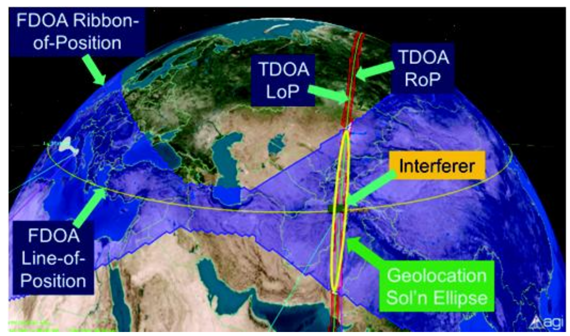


Figure 2. RFI geolocation error for mutually-poor ephemeris errors.

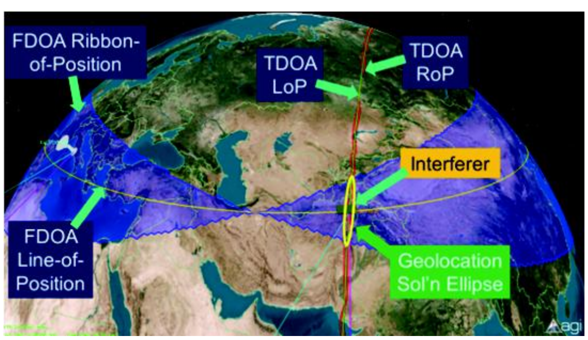


Figure 3. RFI geolocation error for good-v.-poor ephemeris errors.

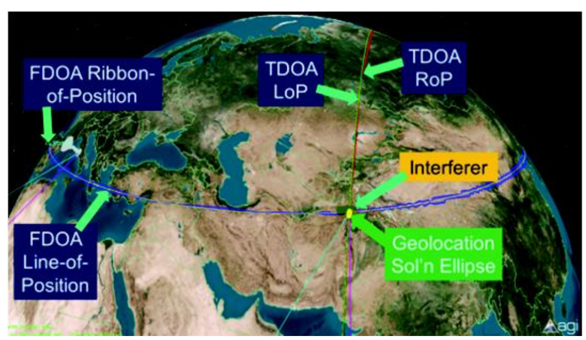


Figure 4. RFI geolocation error for mutually-good ephemeris errors.

For RFI, the relationship between ephemeris error and RFI geolocation error can be illustrated via a sample test case. An interfering emitter was placed in the Middle East region as indicated by the "interferer" position shown in Figure 1. A signal is relayed by two adjacent satellites, respectively, with the downlink station located in south-central Europe. The instantaneous geolocation solution corresponding to this sample case is shown in Figure 2; the intersection of the *time difference of arrival* (TDOA) and *frequency difference of arrival* (FDOA) *Lines of position* (LoPs) shows where the interfering signal is located if the satellites' positions are well-known. The blue and red areas depict the *ribbons of position* (RoPs) of uncertainty that exist based on the satellite 3 σ positional uncertainty of approximately ±20 km in-track, 4 km radial and 2 km cross-track. Figure 3 shows the geolocation error uncertainty which results from a combination of the previous satellite position error uncertainty with the other satellite having a 3 σ positional uncertainty of ±500 m in-track, 100 m radial and 50 m cross-track (reflecting typical operator OD ephemeris precision and accuracy). Figure 4 shows the solution for the case where both satellite ephemerides are known to typical operator precision. These figures show the dramatic impact that accurate and precise ephemerides can have on the RFI mitigation process without having to resort to additional reference emitters and accumulated overlay of multiple geolocation solutions.

INTERPOLATION ERROR ALLOWANCES

The spacecraft operator performs orbit determination on each of its satellites and obtains 3 σ positional uncertainties at the ephemeris initial fit epoch. For spacecraft that are tracked frequently, 3 σ uncertainties could range from fifty to three hundred meters for Geosynchronous Earth Orbit (GEO), one hundred meters for Middle Earth Orbit (MEO) and one hundred meters for Low Earth Orbit (LEO) regime. Estimated uncertainty is included in the ephemerides provided by the external operator, and that uncertainty increases with time.

The ephemeris interpolation required to perform CA and RFI analyses adds error to the extant ephemeris uncertainty. How much error is permissible due solely to interpolation? One can see that if the orbit determination process yielded extremely large uncertainties (*e.g.*, 100 km), it is needless to constrain the step size to maintain the interpolation error to less than, say, 5 meters. Maintaining an unnecessarily small sample size leads to prohibitively large files .

For the satellites of interest in this study, highly tracked space objects would typically have best-case positional accuracies of perhaps 50 meters. Accordingly, 50 meters was selected as an overall precision threshold for interpolation-induced error when drawing conclusions on suitable step size for externally-supplied ephemerides. Because the inaccuracies introduced by ephemeris interpolation would be additive to the original ephemeris uncertainty, as much as 100 meters of positional error could exist by the time the interpolated position is used in CA and RFI computations.

INTERPOLATION METHODS EXAMINED

Interpolation accuracy is now examined when applied to GEO, MEO, LEO and *geosynchronous transfer orbits* (GTO) using several ephemeris interpolation methods, including: 5th-order Lagrange[2,3,4], and 3rd- and 5th-order Hermitian interpolation[5] (both two-point and uniformly-spaced four-point). The benefit of using a dynamic (uneven) step size ephemeris for high-eccentricity orbits will also be examined, as governed by the Sundman transformation for regularized time.

Interpolation Error Metric Approach

Analytic error expressions exist for the Lagrange and Hermitian interpolation methods and are provided in most standard numerical analysis texts.[2, 3] However, these expressions depend entirely upon the node placement/spacing and the function values at each node. The highly variable nature of the computed error when interpolating in Cartesian space indicates that this error metric would have to be repeatedly evaluated for each interpolation condition for all components and then somehow combined on a component-by-component basis to yield the estimated overall positional error.

Instead, for each of the interpolation methods and orbit regimes, the interpolated result is directly compared with a one-second step size "truth" ephemeris (generated using MSISE2000 atmospheric drag, solar radiation pressure, third-body perturbations, and a 25×25 EGM96 gravity field. In doing so, both the precision and accuracy of the interpolation process are assessed (termed "accuracy" for simplicity). The overall distribution of positional error (*i.e.*, truth minus the interpolated result) was then captured at every second throughout the one-day duration. The nature of the underlying distribution is shown in three dimensions in Figure 5; its two-dimensional counterpart is shown in Figure 6.

From these accumulated outcomes, the worst-case and median interpolation errors can be determined. For this study, the ephemeris step size corresponding to the crossing of the adopted allowable 50-meter interpolation error threshold by the worst-case interpolation error trend line was then taken as that interpolation method's performance metric. Although a median (50th percentile) or other percentile metric could have been adopted for this study, the maximum error was selected because the immediate goal is to ensure that the interpolation method works well in all cases.

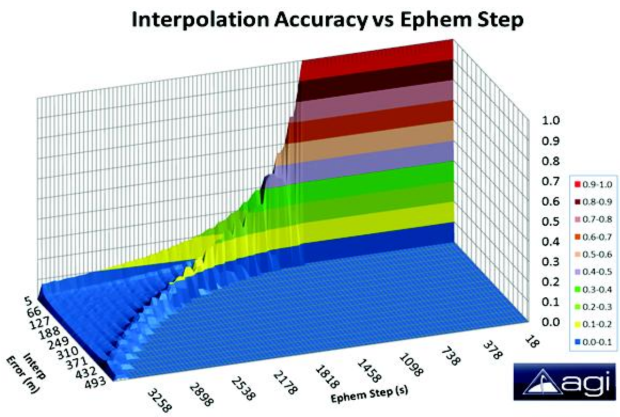


Figure 5. Sample Distribution of Interpolation Error as Function of Ephemeris Step Size (3D)

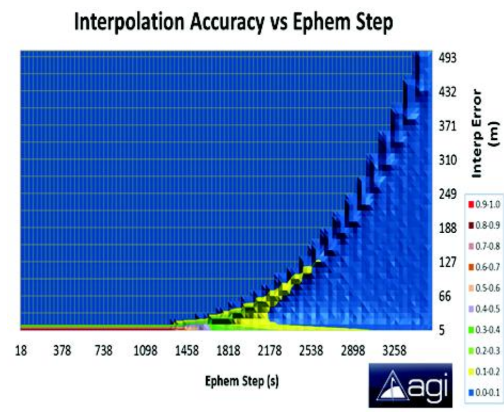
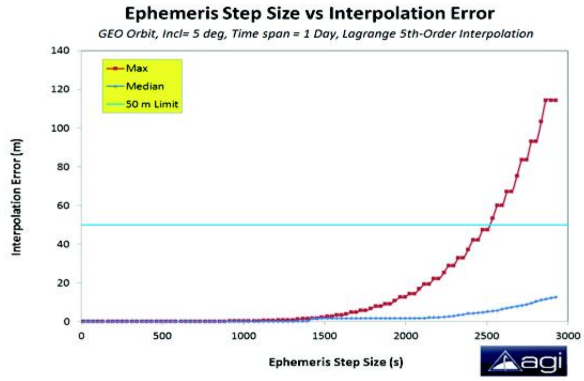


Figure 6. Sample Distribution of Interpolation Error as Function of Ephemeris Step Size (2D)

GEO Regime

The selected interpolation methods were applied to spacecraft in the GEO regime. To acknowledge inclination drift trends for GEO orbits, a 5° orbit inclination is selected for this case.

5th-Order Lagrange Polynomial Interpolation. Lagrange polynomial interpolation is the least accurate of the selected interpolation methods, but it does have several distinct advantages: (1) it only requires position vectors (velocity, acceleration, and jerk are not required); and (2) although it does require n+1 data points for n^{th}-order interpolation, those n+1 points may be non-uniformly spaced. For the GEO orbit regime, interpolation error as a function of ephemeris step size is characterized. Figure 7 and Figure 8 show the dependence upon obtained interpolation precision as a function of ephemeris step size for spacecraft in the GEO regime. The figures indicate that in order to meet the 50-meter accuracy goal, 5th-order Lagrange polynomial interpolation supports a step size not exceeding approximately 2550 seconds (42.5 minutes) for GEO ephemerides. In contrast to other interpolation methods that will follow, the distribution of errors in Figure 8 indicates that the maximum error is a less-common occurrence.



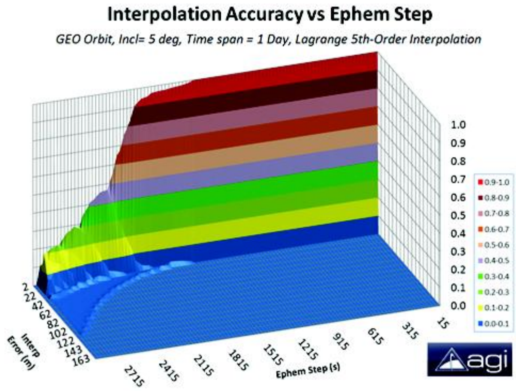
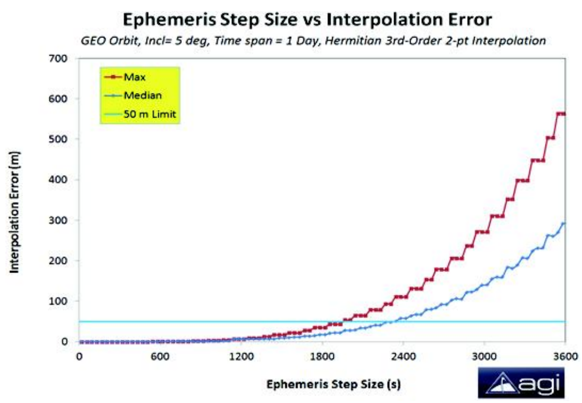


Figure 7. GEO 5th-Order Lagrange Interpolation (Median & Max Error)

Figure 8. GEO 5th-Order Lagrange Interpolation Error Distribution

Two-Point, 3rd-Order Hermitian Interpolation. A variety of Hermitian interpolation orders and methods (equally-spaced, multi-point, two-point, and various orders) are commonly used in astrodynamics. Our first Hermitian method is the simple two-point Hermitian 3rd-order interpolation scheme. Because it only requires position and velocity for the two bounding points, this approach is well suited for unequally spaced ephemerides. Figure 9 and Figure 10 indicate that in order to meet the 50-meter accuracy goal, two-point Hermitian 3rd-order interpolation supports a step size not exceeding approximately 1800 seconds (30 minutes) for GEO ephemerides.



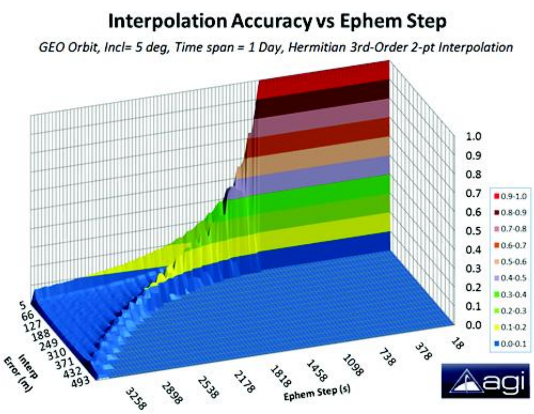
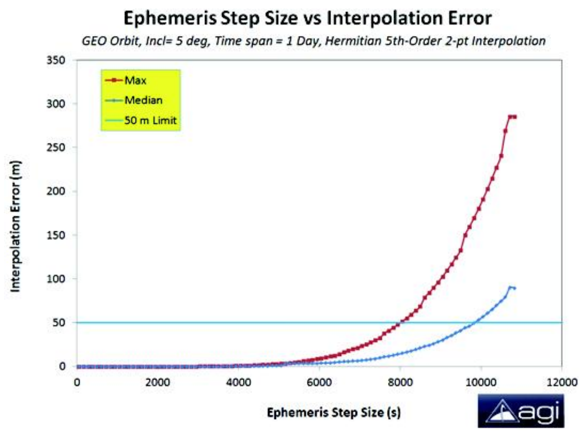


Figure 9. GEO 2-Point 3rd-Order Hermitian Interpolation (Median & Max Error)

Figure 10. GEO 2-Point 3rd-Order Hermitian Interpolation Error Distribution

Two-Point, 5th-Order Hermitian Interpolation. In addition to the position and velocity vector ephemeris data required by the Hermitian 3rd-order method, this method requires the addition of acceleration vector data. Again, because this method only requires ephemeris data at two bounding points, it is well suited for unequally spaced ephemerides. Figure 11 and Figure 12 indicate that for the 50-meter accuracy goal, two-point 5th-order Hermitian interpolation supports a step size as large as 8000 seconds (well over two hours) for GEO ephemerides.



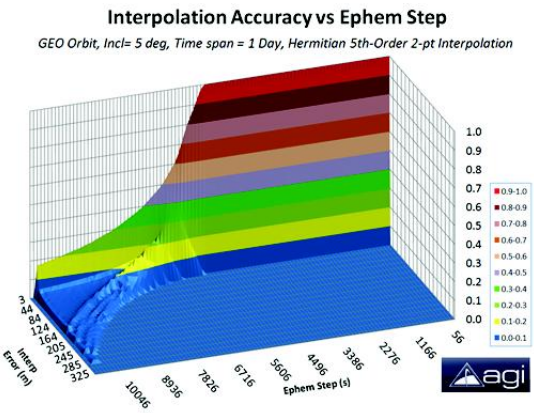
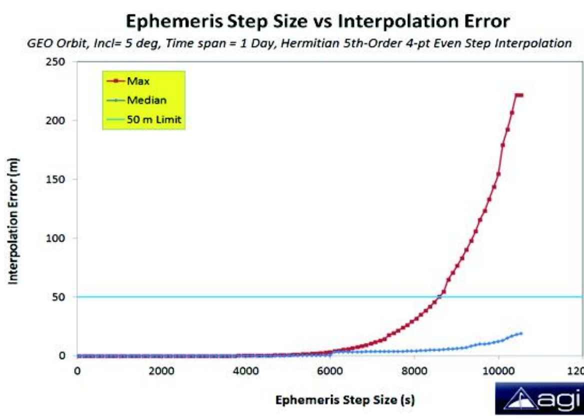


Figure 11. GEO 2-Point 5th-Order Hermitian Interpolation (Median & Max Error)

Figure 12. GEO 2-Point 5th-Order Hermitian Interpolation Error Distribution

It is concluded that even though this method requires the ephemeris provider to add three additional parameters to the data (a 30% increase in the number of columns), the actual file size can actually shrink by a factor of more than three for the same interpolation accuracy since this interpolation method permits substantially larger step sizes for the same interpolation accuracy as Hermitian 3rd-order interpolation.

Four Evenly-Spaced Points, 5th-Order Hermitian Interpolation. While this method only requires position and velocity data similar to the Hermitian 3rd-order method, this method requires that the ephemeris points be evenly spaced in time. This makes this method ill-suited for the general case where unequally spaced ephemerides may be provided. Figure 13 and Figure 14 indicate that for the 50-meter accuracy goal, four-point 5th-Order Hermitian interpolation supports a step size as large as 8600 seconds (more than two hours) for GEO ephemerides.



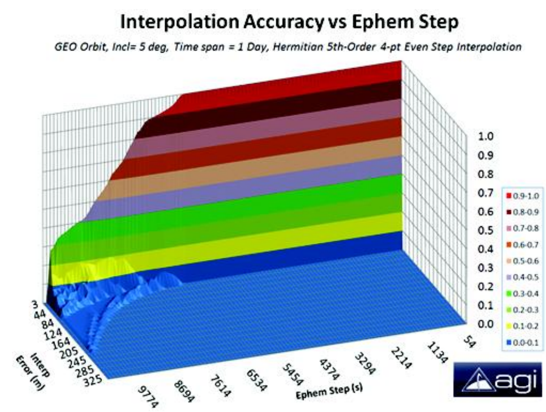


Figure 13. GEO 4-Point Even Step 5th-Order Hermitian Interpolation (Median & Max Error)

Figure 14. GEO 4-Point Even Step 5th-Order Hermitian Interpolation Error Distribution

As one would expect, the accuracy of this approach is virtually identical to that obtained using the two-point 5th-order Hermitian interpolation method. That it can accommodate slightly larger step sizes (up to ten minutes longer than the two-point 5th-order Hermitian interpolation method) can be attributed to its ability to fit a non-linear acceleration profile between bracketing points by using four points. This four-point method has the advantage that files can be 30% smaller than for the two-point 5th-order Hermitian interpolation method without substantial loss of accuracy, but as will be demonstrated later, the requirement for even step size prevents implementation of best practices for accommodating ephemeris discontinuities in position, velocity and/or acceleration.

MEO Regime

The selected interpolation methods were applied to spacecraft in the MEO regime. For this regime, a 26578 km circular orbit inclined at 55° (GPS orbit) was selected.

5th-Order Lagrange Polynomial Interpolation. Figure 15 and Figure 16 indicate that in order to meet the 50-meter accuracy goal, 5th-order Lagrange polynomial interpolation supports a step size not exceeding approximately 1350 seconds (22.5 minutes) for MEO ephemerides.

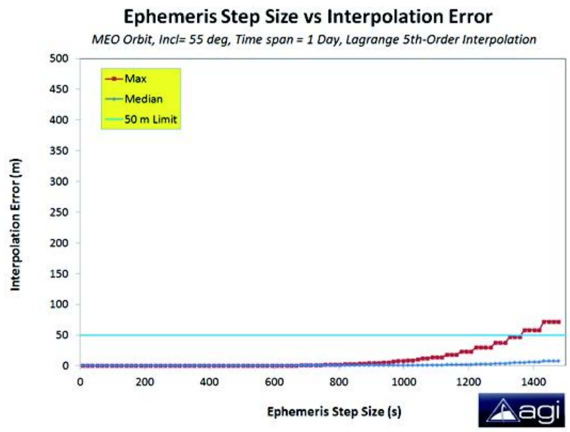


Figure 15. MEO 5th-Order Lagrange Interpolation (Median & Max Error)

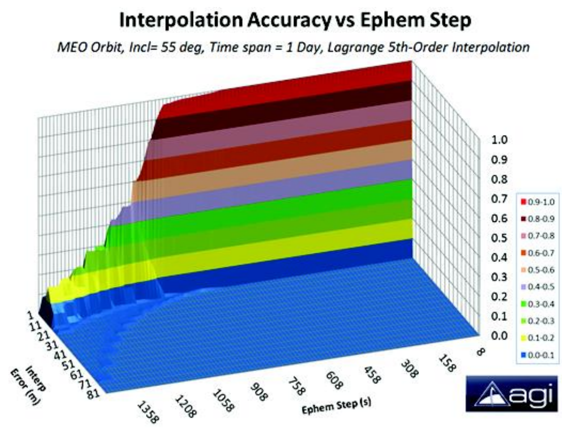


Figure 16. MEO 5th-Order Lagrange Interpolation Error Distribution

Two-Point, 3rd-Order Hermitian Interpolation. Figure 17 and Figure 18 indicate that in order to meet the 50-meter accuracy goal, two-point Hermitian 3rd-order interpolation supports a step size not exceeding approximately 1038 seconds (17 minutes) for MEO ephemerides.

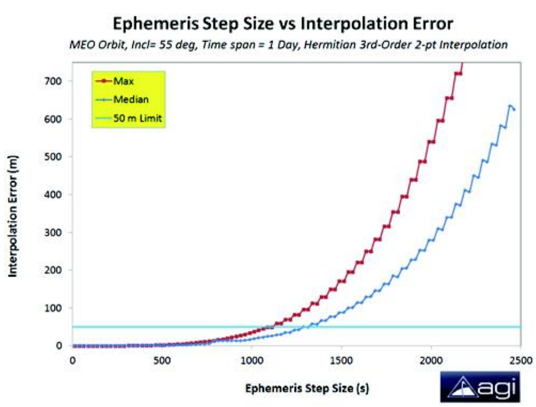


Figure 17. MEO 2-Point 3rd-Order Hermitian Interpolation (Median & Max Error)

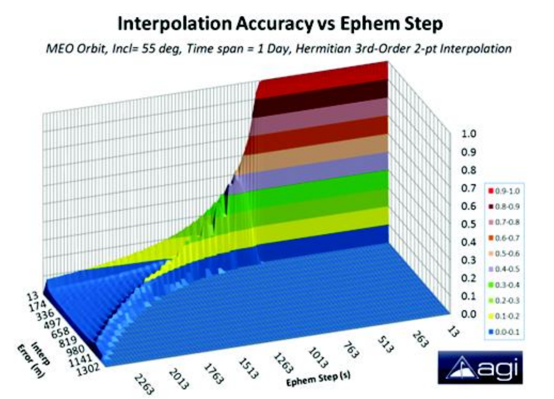
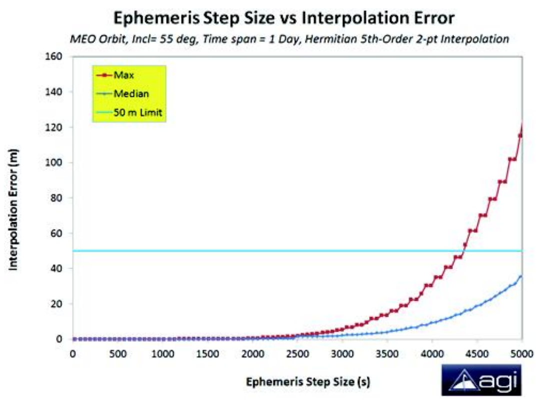


Figure 18. MEO 2-Point 3rd-Order Hermitian Interpolation Error Distribution

Two-Point, 5th-Order Hermitian Interpolation. Figure 19 and Figure 20 indicate that for the 50-meter accuracy goal, two-point 5th-Order Hermitian interpolation supports a step size as large as 4350 seconds (72.5 minutes) for MEO ephemerides.



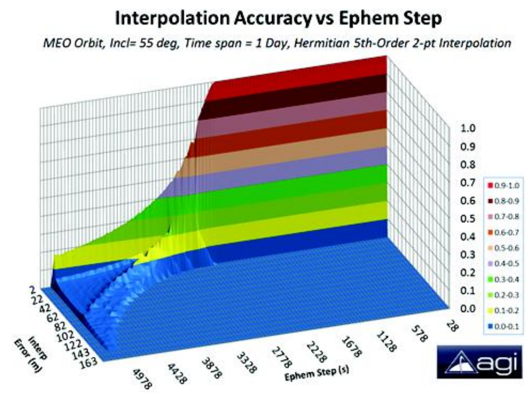
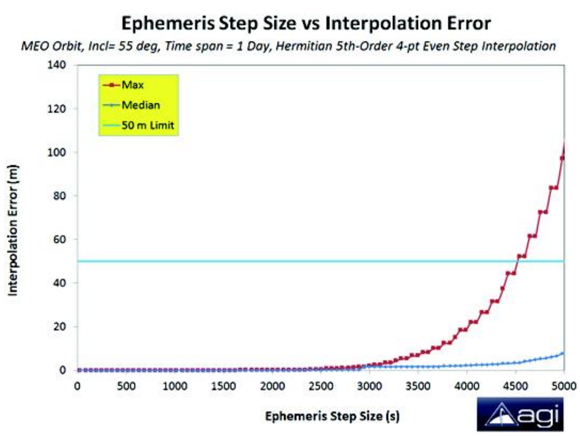


Figure 19. MEO 2-Point 5th-Order Hermitian Interpolation (Median & Max Error)

Figure 20. MEO 2-Point 5th-Order Hermitian Interpolation Error Distribution

As in the GEO case, it can easily be concluded that the actual file size can shrink by a factor of more than three compared to the Hermitian 3rd-order interpolation for the same interpolation accuracy.

Four Evenly-Spaced Points, 5th-Order Hermitian Interpolation. Figure 21 and Figure 22 indicate that for the 50-meter accuracy goal, four-point 5th-order Hermitian interpolation supports a step size as large as 4538 seconds (over 75 minutes) for MEO ephemerides. As in the GEO test case, the accuracy of this approach is similar to that obtained using the two-point 5th-order Hermitian interpolation method.



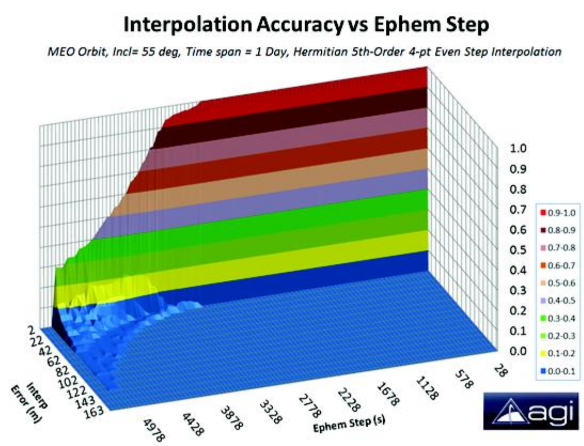


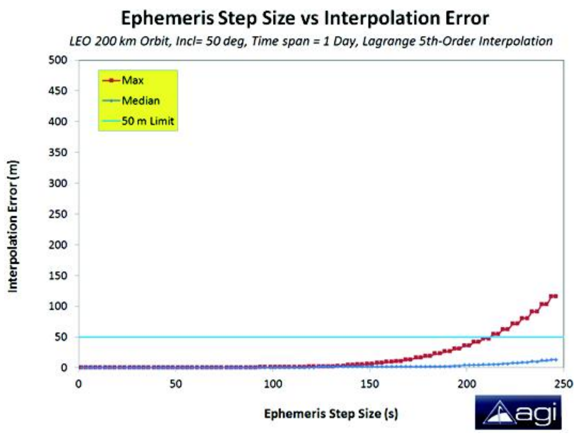
Figure 21. MEO 4-Point Even Step 5th-Order Hermitian Interpolation (Median & Max Error)

Figure 22. MEO 4-Point Even Step 5th-Order Hermitian Interpolation Error Distribution

LEO Regime

The selected interpolation methods were applied to spacecraft in the LEO regime. In order to fully stress the interpolation method accuracy, a 6578 km circular orbit inclined at 50° is selected for this test case.

5th-Order Lagrange Polynomial Interpolation. Figure 23 and Figure 24 indicate that in order to meet the 50-meter accuracy goal, 5th-order Lagrange polynomial interpolation supports a step size not exceeding approximately 220 seconds (3.7 minutes) for LEO ephemerides.



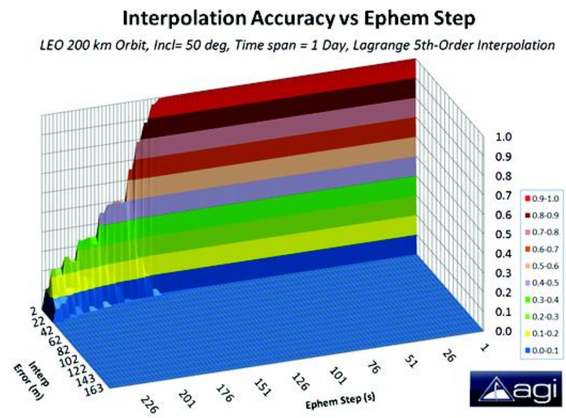
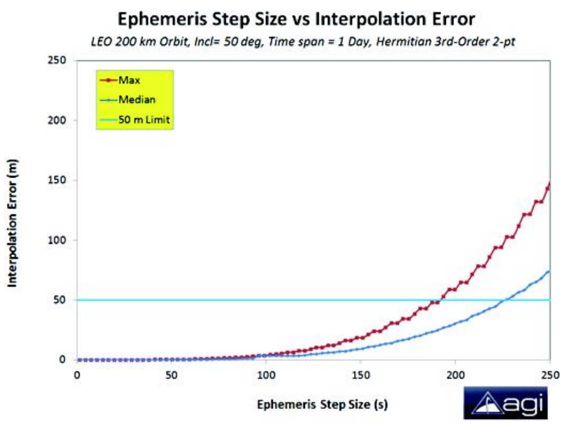


Figure 23. LEO 5th-Order Lagrange Interpolation (Median & Max Error)

Figure 24. LEO 5th-Order Lagrange Interpolation Error Distribution

Two-Point, 3rd-Order Hermitian Interpolation. Figure 25 and Figure 26 indicate that in order to meet the 50-meter accuracy goal, two-point Hermitian 3rd-order interpolation supports a step size not exceeding approximately 180 seconds (3 minutes) for LEO ephemerides.



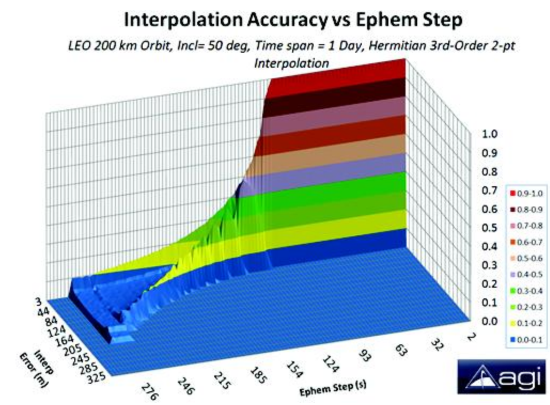
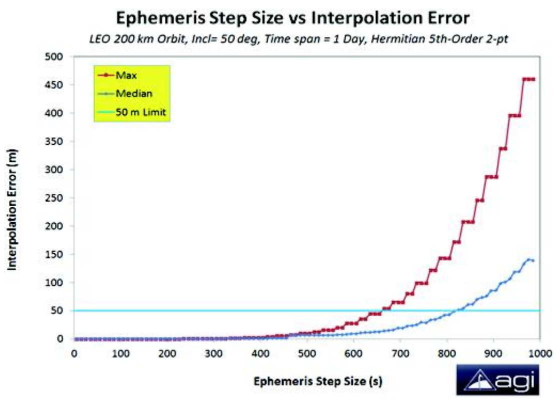


Figure 25. LEO 2-Point 3rd-Order Hermitian Interpolation (Median & Max Error)

Figure 26. LEO 2-Point 3rd-Order Hermitian Interpolation Error Distribution

Two-Point, 5th-Order Hermitian Interpolation. Figure 27 and Figure 28 indicate that for the 50-meter precision goal, two-point 5th-Order Hermitian interpolation supports a step size as large as 660 seconds (eleven minutes) for LEO ephemerides.



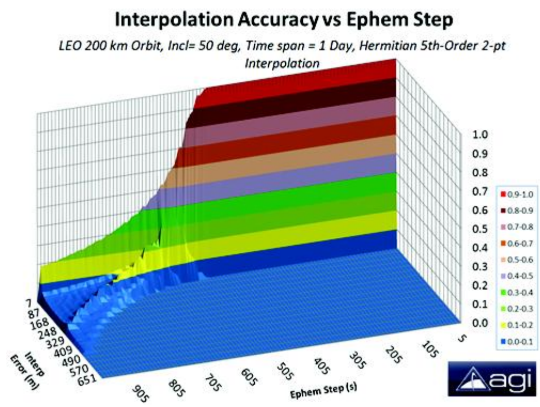
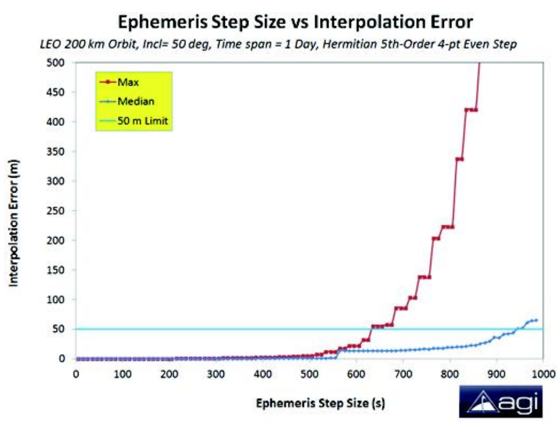


Figure 27. LEO 2-Point 5th-Order Hermitian Interpolation (Median & Max Error)

Figure 28. LEO 2-Point 5th-Order Hermitian Interpolation Error Distribution

As in the GEO case, it can easily be concluded that the actual file size can shrink by a factor of more than three compared to the Hermitian 3rd-order interpolation for the same interpolation accuracy.

Four Evenly-Spaced Points, 5th-Order Hermitian Interpolation. Figure 29 and Figure 30 indicate that for the 50-meter accuracy goal, four-point 5th-Order Hermitian interpolation supports a step size as large as 645 seconds (almost eleven minutes) for LEO ephemerides.



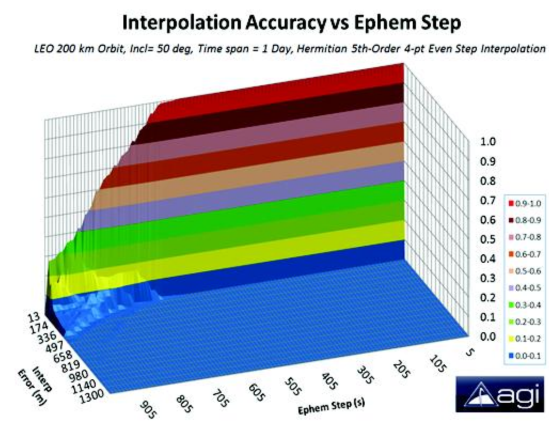


Figure 29. LEO 4-Point Even Step 5th-Order Hermitian Interpolation (Median & Max Error)

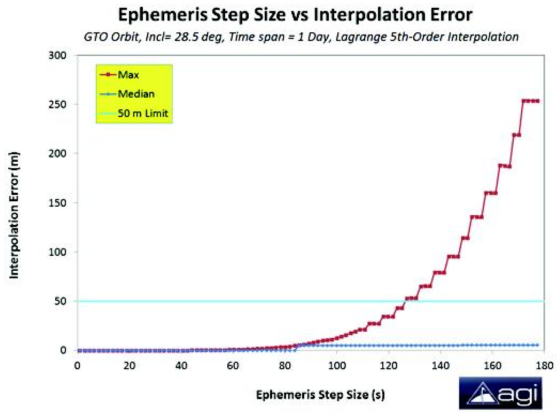
Figure 30. LEO 4-Point Even Step 5th-Order Hermitian Interpolation Error Distribution

As in the GEO test case, the accuracy of this approach is similar to that obtained using the two-point 5th-order Hermitian interpolation method.

GTO Regime

The selected interpolation methods were applied to spacecraft in the GTO regime. For these tests, a spacecraft in a 6578 x 42164 km Geosynchronous Transfer Orbits (GTO) was selected. Since launches to this orbit are common from Cape Canaveral, a 28.5° orbit inclination is selected.

5th-Order Lagrange Polynomial Interpolation. Figure 31 and Figure 32 indicate that in order to meet the 50-meter accuracy goal, 5th-Order Lagrange polynomial interpolation supports a step size not exceeding approximately 125 seconds (2 minutes) for GTO ephemerides.



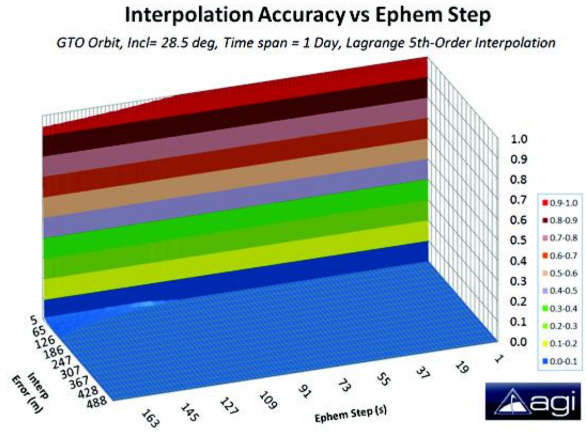
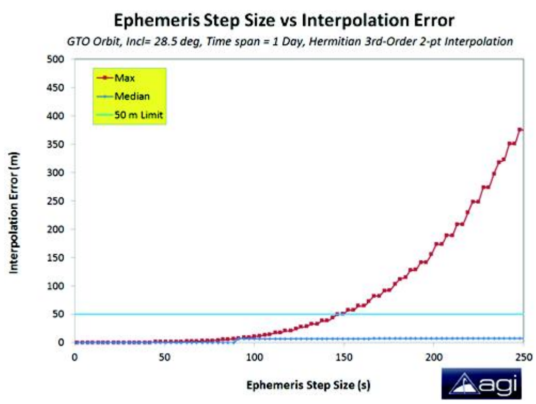


Figure 31. GTO 5th-Order Lagrange Interpolation (Median & Max Error)

Figure 32. GTO 5th-Order Lagrange Interpolation Error Distribution

Two-Point, 3rd-Order Hermitian Interpolation. Figure 33 and Figure 34 indicate that in order to meet the 50-meter accuracy goal, two-point Hermitian 3rd-order interpolation supports a step size not exceeding approximately 150 seconds (2.5 minutes) for GTO ephemerides.



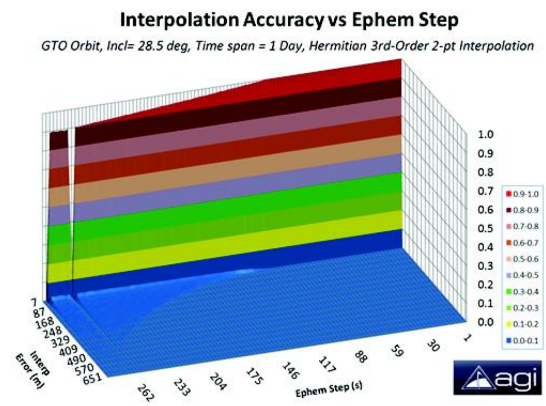
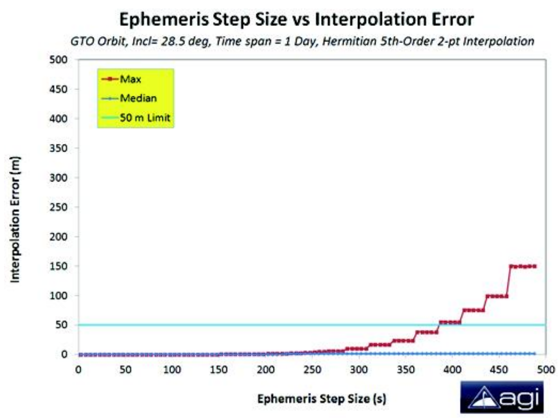


Figure 33. GTO 2-Point 3rd-Order Hermitian Interpolation (Median & Max Error)

Figure 34. GTO 2-Point 3rd-Order Hermitian Interpolation Error Distribution

Two-Point, 5th-Order Hermitian Interpolation. Figure 35 and Figure 36 indicate that for the 50-meter accuracy goal, two-point 5th-Order Hermitian interpolation supports a step size as large as 390 seconds (6.5 minutes) for GTO ephemerides.



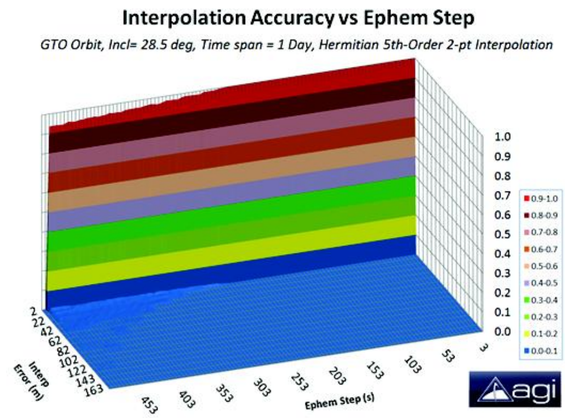
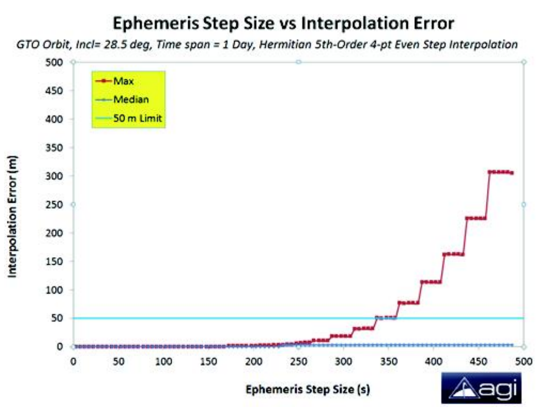


Figure 35. GTO 2-Point 5th-Order Hermitian Interpolation (Median & Max Error)

Figure 36. GTO 2-Point 5th-Order Hermitian Interpolation Error Distribution

As in the GEO case, it can easily be concluded that the actual file size can shrink by a factor of more than three compared to the Hermitian 3rd-order interpolation for the same interpolation accuracy.

Four Evenly-Spaced Points, 5th-Order Hermitian Interpolation. Figure 37 and Figure 38 indicate that for the 50-meter accuracy goal, four-point 5th-Order Hermitian interpolation supports a step size as large as 340 seconds (5.5 minutes) for GTO ephemerides.



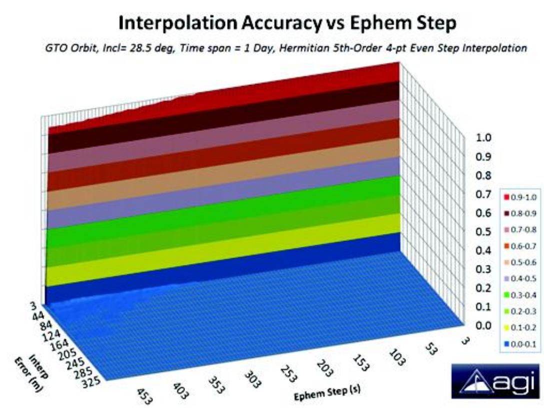


Figure 37. GTO 4-Point Even Step 5th-Order Hermitian Interpolation (Median & Max Error)

Figure 38. GTO 4-Point Even Step 5th-Order Hermitian Interpolation Error Distribution

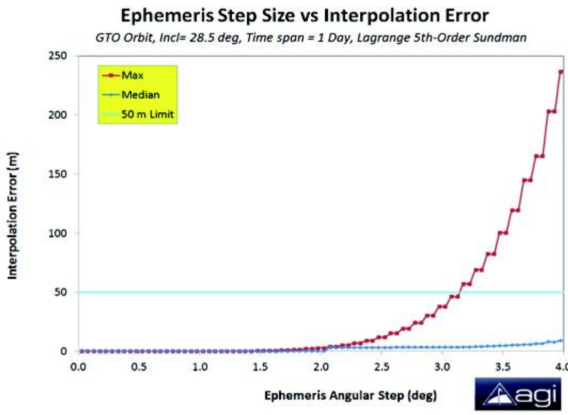
As in the GEO test case, the accuracy of this approach is similar to that obtained using the two-point 5th-order Hermitian interpolation method. Notably, however, it is worse in both this GTO case and the previous LEO case. The authors conjecture that perhaps the rapid change in acceleration due to the LEO and GTO cases is best accommodated by not attempting to fit through more than one time interval.

EPHEMERIS STEP SIZE CONTROL VIA REGULARIZED TIME

It has been demonstrated that interpolation for circular orbits exhibits good, uniform performance. However, the GTO case has shown that highly-eccentric orbits present a problem for all interpolators considered and that small step sizes must be used to ensure that the overall accuracy goal of 50 meters is met. But as noted earlier, only one of the four examined interpolation methods (Four-Point Even Step 5th-Order Hermitian) requires the time step size to be constant throughout the ephemeris. The other three methods have no such restriction. Accordingly, the well-known concept of "regularized time" (using the Sundman transformation) is examined.[6]

5th-Order Lagrange Polynomial Interpolation

Figure 39 and Figure 40 indicate that in order to meet the 50-meter accuracy goal, 5th-order Lagrange polynomial interpolation supports an angular step size not exceeding approximately 3.2° for GTO ephemerides using a regularized time approach.



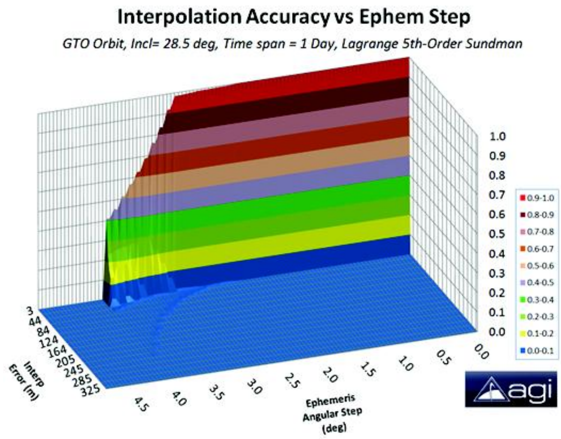
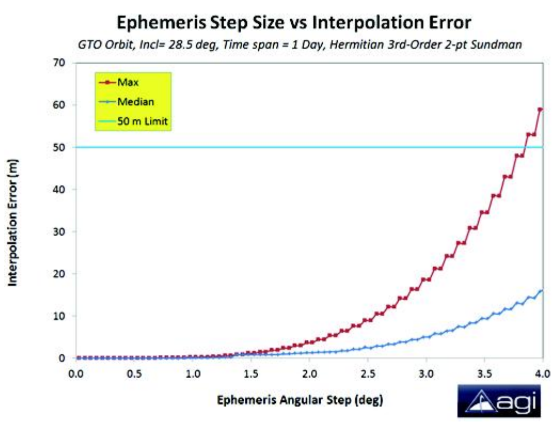


Figure 39. GTO Sundman 5th-Order Lagrange Interpolation (Median & Max Error)

Figure 40. GTO Sundman 5th-Order Lagrange Interpolation Error Distribution

Two-Point, 3rd-Order Hermitian Interpolation

Figure 41 and Figure 42 indicate that in order to meet the 50-meter accuracy goal, two-point Hermitian 3rd-order interpolation supports an angular step size not exceeding approximately 3.8° for GTO ephemerides using a regularized time approach.



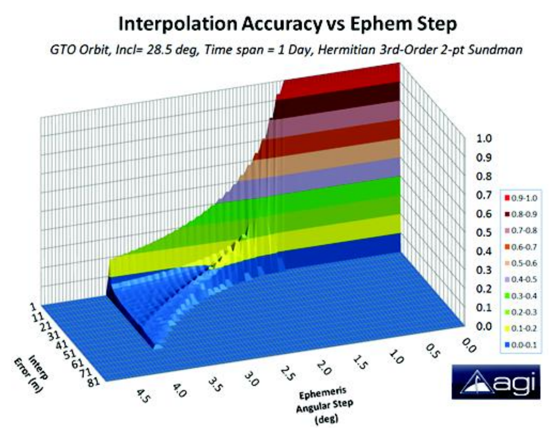


Figure 41. GTO Sundman 2-Point 3rd-Order Hermitian Interpolation (Median & Max Error)

Figure 42. GTO Sundman 2-Point 3rd-Order Hermitian Interpolation Error Distribution

Two-Point, 5th-Order Hermitian Interpolation

Figure 43 and Figure 44 indicate that for the 50-meter accuracy goal, two-point 5th-Order Hermitian interpolation supports an angular step size not exceeding approximately 10.5° for GTO ephemerides using a regularized time approach.

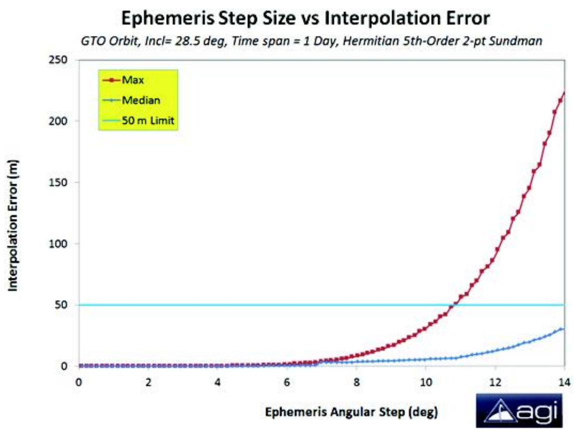


Figure 43. GTO Sundman 2-Point 5^{th}-Order Hermitian Interpolation (Median & Max Error)

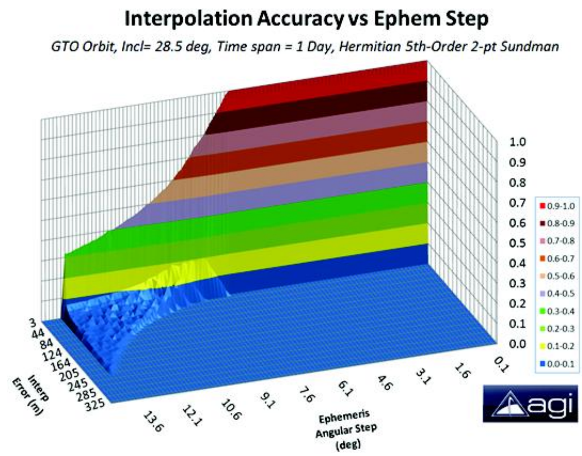


Figure 44. GTO Sundman 2-Point 5^{th}-Order Hermitian Interpolation Error Distribution

As in the GEO case, it can easily be concluded that the actual file size can shrink by a factor of more than three compared to the Hermitian 3^{rd}-order interpolation for the same interpolation accuracy using a regularized time approach.

ADDITIONAL DISCUSSION AND RECOMMENDED PRACTICES

Above results notwithstanding, an important consideration is the precision of the supplied ephemeris. Another important factor is how to accommodate discontinuities in the ephemeris. Discontinuities can occur in spacecraft position (due to ephemeris "joining"), velocity (due to impulsive maneuver modeling, and acceleration (due to perturbations step functions). Two best practices that handle such uncertainties are to (1) use a two-point interpolation method that accommodates unevenly-spaced ephemeris points, and (2) ensure that the ephemeris includes positional information at each discontinuity boundary.

"Discontinuities" in position and/or velocity can and do occur when two ephemerides are joined together. When interpolating across the former boundary, a small Gibbs' effect (ringing) due to under-sampling has been observed in the resultant values relative to the true ephemeris. Interpolation methods that require more than two node points are likely to be more susceptible to this issue. Such an effect can then manifest itself in analysis end-products (*e.g.*, artifacts in the sea surface topography of altimetric missions). One method for removing such artifacts is to overlap the two ephemerides (*e.g.*, one orbit revolution of overlap) to be joined and then employ a "smoothing function" to compute a weighted average on each side of the terminus.

Fortunately for high-fidelity spacecraft motion modeling, natural discontinuities are not typically seen until the second derivative of position (acceleration). These discontinuities can arise from the introduction of finite burns, step-function change to solar perturbations (i.e., umbral crossings) or drag forces (e.g., deorbit device inflation). The Hermitian interpolation methods employ a linear variation to second derivatives (acceleration), and if such discontinuities occur in between the supplied ephemeris time points, this will undoubtedly affect interpolation performance.

One should also consider the time scale used as the independent variable in the ephemeris. For example, time intervals between UTC epochs are not necessarily uniform in the presence of a leap second. Using a time reference such as International Atomic Time (TAI) or other uniform time scales can circumvent this problem.

Another important consideration is the “terminal effect” which occurs when trying to interpolate at the beginning or end of the supplied ephemeris. “Terminal effects” denote that the error is larger at the ends of the ephemeris, usually by an order of magnitude, because the information at ephemeris endpoints is one-sided. This effect may be mitigated by relying upon two-point interpolation schemes.

STEP SIZE “RULES-OF-THUMB”

Previously, it has been advocated that ephemeris step size can be selected using several rules-of-thumb, as follows[7]:

> It is important that each ephemeris table be created so that interpolation of the table differs little from the value that would have been computed using SGP4 with the TLE directly. Typically, 90 points per orbit is needed for adequate interpolation of nearly circular orbits. For eccentric orbits, it is critical to have enough samples near perigee to do accurate interpolation. A general rule is to use a step size that would produce 90 points per orbit for a circular orbit at the same perigee as the eccentric orbit. A maximum step of 300 seconds was also adopted. Thus, LEOs generally use 60 sec steps while GEOs typically use 300 sec steps.

Some similarly rough, method-specific rules-of-thumb may now be developed based upon the above findings. The results of this study can be captured as shown in Table 1, Table 2, Table 3, and Table 4.

.

Table 1. Maximum Step Size Meeting Accuracy Goal by Method & Orbit Regime (seconds).

Orbit Regime:	LEO	GTO	MEO	GEO	GTO w/Sundman
Lagrange 5th-Order	220	126	1350	2550	N/A
Hermitian 3rd-Order 2-pt	191	149	1038	1950	N/A
Hermitian 5th-Order 2-pt	655	393	4350	7937	N/A
Hermitian 5th-Order 4-pt Even Step	655	348	4538	8586	N/A

Table 2. Maximum Angular Step Meeting Accuracy Goal by Method & Orbit Regime (degrees).

Orbit Regime:	LEO	GTO	MEO	GEO	GTO w/Sundman
Orbit Period (sec)	*5310*	*37981*	*43122*	*86164*	*37981*
Lagrange 5th-Order	14.9	N/A	11.3	10.7	3.2
Hermitian 3rd-Order 2-pt	12.9	N/A	8.7	8.1	3.8
Hermitian 5th-Order 2-pt	44.4	N/A	36.3	33.2	10.9
Hermitian 5th-Order 4-pt Even Step	44.4	N/A	37.9	35.9	N/A

Table 3. Number Steps Per Orbit Required by Method & Orbit Regime.

Orbit Regime:	LEO	GTO	MEO	GEO	GTO w/Sundman
Lagrange 5th-Order	25	302	32	34	113
Hermitian 3rd-Order 2-pt	28	255	42	45	95
Hermitian 5th-Order 2-pt	9	97	10	11	33
Hermitian 5th-Order 4-pt Even Step	9	110	10	11	N/A

Table 4. Approximate File Size for Two-Week Ephemeris by Method & Orbit Regime (Kb).

Orbit Regime:	LEO	GTO	MEO	GEO	GTO w/Sundman
Lagrange 5th-Order	650	1136	106	56	424
Hermitian 3rd-Order 2-pt	749	960	138	73	204
Hermitian 5th-Order 2-pt	312	520	47	26	71
Hermitian 5th-Order 4-pt Even Step	218	411	32	17	N/A

The following observations can be made:

1. The original “rules-of-thumb” are appropriately conservative and in-family with our observed empirical observations for the Lagrange 5^{th}-order interpolation scheme;
2. Ephemeris file size can be reduced by a factor of seven for highly-eccentric orbits when higher-fidelity interpolation scheme is combined with regularized time;

The original rule of “90 points” per circular orbit corresponding to perigee altitude fails to capture the rapid motion at perigee. This can be readily seen by computing the circular orbit velocity for our LEO case (7.78 km/s) versus the perigee velocity for our GTO case (10.24 km/s)

EPHEMERIS NUMERICAL PRECISION

Even the best interpolation method will not work well if based on insufficient information. The authors have observed an analyst tendency to select a limited numerical precision without considering its impact on ephemeris usability or derived interpolation accuracy.

Figure 45 and Figure 46 examine, for each of the principle interpolation methods employed in this study, how the ephemeris step size which yields the maximum allowable interpolation error of 50 meters varies as a function of ephemeris parameter precision. In these figures, the position of each of the three Cartesian components of position and velocity was rounded to the nearest unit of precision shown in the bottom categories and then the step size which yielded a positional accuracy of 50 meters was determined for each precision bin.

AGI’s Center for Space Standards and Innovation (CSSI) has advocated that ephemerides be formatted such that time is specified to the nearest millisecond, position to a precision better than a millimeter, and velocity to better than 10^{-6} m/s. It can be observed that CSSI’s recommended precision specifications ensure that all interpolation methods easily meet the interpolation accuracy goals. It should be noted that decreasing precision even four places from the CSSI recommendation was sufficient to give several of the interpolation methods difficulty.

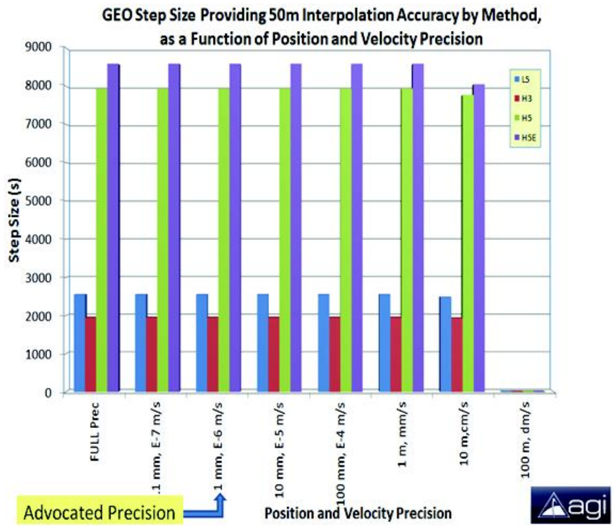


Figure 45. GEO Step Size Ensuring 50 m Interpolation Accuracy v. Ephemeris Precision

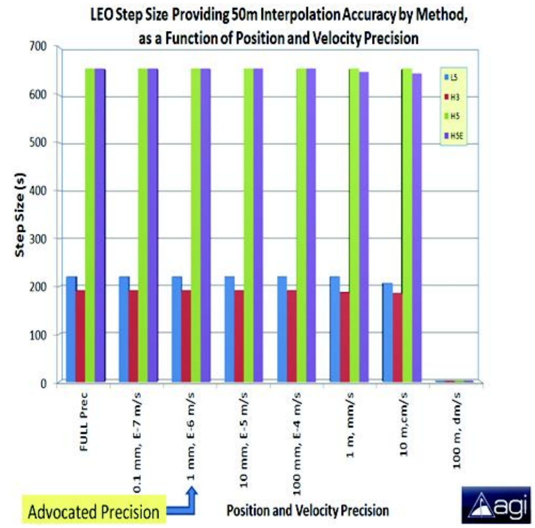


Figure 46. LEO Step Size Ensuring 50 m Interpolation Accuracy v. Ephemeris Precision

COVARIANCE INTERPOLATION

It is a common analysis requirement to be able to interpolate covariance matrices within the satellite ephemeris files. While it is tempting to simply perform a linear or polynomial fit of each covariance matrix element, such an approach likely will produce non-symmetric, negative-definite (invalid) covariance matrices that yield non-orthogonal ellipsoids. Fortunately, a number of suitable alternative methods have been devised to circumvent this problem. These methods consist of two basic approaches.

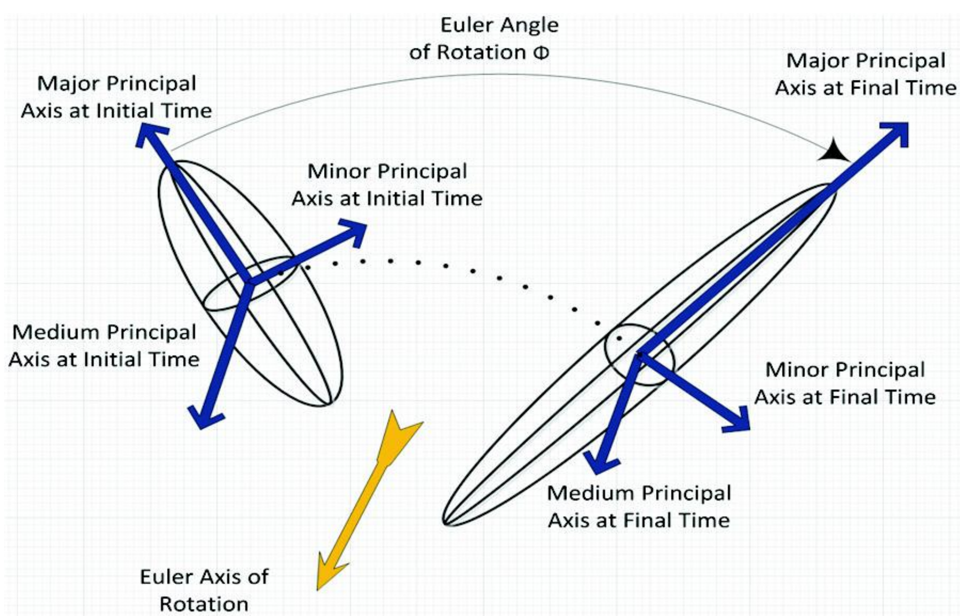


Figure 47. Graphical Depiction of *Eigenmorphing* Covariance Interpolation

“Eigenmorphing”

In the first covariance interpolation approach, a weighted-average blending between two bounding nodes can be accomplished.[8, 9] The authors have coined the phrase *eigenmorphing* for one such technique. First, the covariance matrices on either side of the time point of interest are subjected to eigenvector/eigenvalue decomposition, and the eigenvalues (ellipsoid semi-axes) are linearly transitioned across the interpolation step. The Euler axis and angle corresponding to the mapping from the initial to the final principle axes can then be determined. Then, by varying the eigenvectors smoothly about the Euler axis across the determined angle for this same step, the interpolated covariance may be reconstructed by pre- and post- multiplication of the matrix of eigenvectors with the diagonal matrix of eigenvalues. As shown in Figure 47, this results in smooth, realistic motion of the ellipsoid along the trajectory, which is appropriate for conjunction detection and visualization.

State Transition

In the second covariance interpolation approach, the state transition function can either be propagated from an integration bounding node or interpolated between the integration nodes to the time of interest.[10] This state transition function can then map the covariance from an integration node to the requested time. This method has the disadvantage that it may not adequately account for additive process noise that actually occurs during that time interval because the state transition force model may omit important forces (maneuvers, drag, solar radiation pressure, etc.). But this method does have the advantage that the computationally costly eigenvalue/eigenvector decomposition process can be avoided.

While this latter approach does work well in practice, there are several assumptions and limitations. The eigenvectors are assumed to not rotate more than 90° between interpolation end-points. The designations of major, medium and minor principle axes, although dynamically moving, remain the same between the end-points. The user must also check for “flipped” principle axis directions (i.e. 180° away). Finally, while the velocity covariance can be similarly decomposed and interpolated, an interpolation method for the off-diagonal elements of a 6x6 covariance matrix that link velocity and position uncertainties is not suitably addressed.

CONCLUSIONS

Ephemeris requirements, including ephemeris numerical precision and interpolation accuracy from a variety of interpolation methods, has been explored. For collision avoidance and radio-frequency-interference analyses , the end goal is to minimize the positional error between each supplied ephemeris point. From that perspective, the Lagrange 5$^{\underline{th}}$-order and Hermitian 3$^{\underline{rd}}$-Order methods give acceptable performance, while the 5$^{\underline{th}}$-order methods provided very good performance. If velocity information is not available, the standard Lagrange interpolation method is suitable as long as the ephemeris step size is sufficiently reduced. For highly-elliptical orbits, the non-uniform ephemeris point spacing based upon the Sundman transformation (regularized time) is a good way to minimize ephemeris file size (by as much as a factor of eight) while maintaining the desired accuracy.

The user must decide what error metrics and working aspects of interpolation are best-suited to their needs when selecting an interpolation method. For this study, maximum error was adopted.

Rather than adopting a single interpolation method or approach, a rules-based interpolation hierarchy may be worthy of consideration. For example, the authors have also had good success in the past designing an altitude-based table containing specified polynomial degrees (or alternatively interpolation methods) that best minimize the maximum errors.

FUTURE WORK

Ephemeris and interpolation accuracy aspects potentially worthy of study include: (1) examination of interpolation accuracy across a variety of position, velocity and acceleration domains; (2) interpolation in different reference frames or orbital element sets; (3) interpolation across discontinuities; and (4) examination of more interpolation methods (e.g. taut splines, bi-cubic, Least-Sum-Squares quadratic interpolation or higher-order Lagrange and Hermitian interpolations).

ACKNOWLEDGMENTS

The authors wish to thank Jim Woodburn, David Finkleman, and Sal Alfano for their insightful comments and review of this paper.

REFERENCES

[1] Consultative Committee for Space Data Systems (2009), *Orbit Data Messages*, CCSDS 502.0-B-2 Blue Book, Recommended Standard Issue 2, Washington DC, November 2009.

[2] Chapra, S.C., and R.P. Canale (1985), *Numerical Methods for Engineers*. McGraw-Hill Book Company (ISBN 0-07-010664-9).

[3] Press, W.H., B.P. Flannery, S.A. Teukolsky, W.T. Vetterling (1992), *Numerical Recipes in C: The Art of Scientific Computing, 2nd Ed*. Cambridge University Press, Cambridge (ISBN 0521431085).

[4] Berrut, J.P. and L.N. Trefethen (2004), "Barycentric Lagrange Interpolation." SIAM Review, Vol. 46, No. 3. pp. 501-517.

[5] Johnson, L.W. (1982), *Numerical Analysis 2nd ed.*, Addison-Wesley Publishing (ISBN 0201103923).

[6] Berry, M., L. Healy (2002), "The Generalized Sundman Transformation for Propagation of High-Eccentricity Elliptical Orbits." Paper AAS 02-109, from Alfriend, K.T., *et al*. (eds.), *Spaceflight Mechanics 2002 - Advances in the Astronautical Sciences, Vol. 112, Part I,* Proceedings of the AAS/AIAA Space Flight Mechanics Meeting, San Antonio, Texas, January 27-30, 2002, pp. 127-46.

[7] Coppola, V., S. Dupont, K. Ring, and F. Stoner, (2009) "Assessing Satellite Conjunctions for the Entire Space Catalog using COTS Multi-core Processor Hardware." Pittsburgh: Paper AAS 09-374 from Rao, *et al*. (eds.), *Astrodynamics 2009 - Advances in the Astronautical Sciences, Vol. 112, Part II,* Proceedings of the AAS/AIAA Astrodynamics Specialist Conference, Pittsburgh, Pennsylvania, August 9-13, 2009, pp. 1193-1206 (ISBN 9780877035572).

[8] Gist, R.G., and D.L. Oltrogge (1999) "Collision Vision: Covariance Modeling and Intersection Detection for Spacecraft Situational Awareness." Paper AAS 99-351 from Howell, *et al*. (eds.), *Astrodynamics 1999 - Advances in the Astronautical Sciences, Vol. 103, Part I,* Proceedings of the AAS/AIAA Astrodynamics Specialist Conference, Girdwood, Alaska, August 16-19, 2009, pp. 783-796.

[9] Woodburn, J.W., S. Tanygin (2002), "Position Covariance Visualization." Paper AIAA 2002-4985, Proceedings of the AIAA/AAS Astrodynamics Specialist Conference and Exhibit, Monterey, California, August 5-8, 2002.

[10] Alfano, S. (2004), "Orbital Covariance Interpolation," Paper AAS 04-223 from Coffey, S.L., *et al*. (eds.), *Spaceflight Mechanics 2004 – Advances in the Astronautical Sciences*, *Vol. 119, Part II*, Proceedings of the 14th AAS/AIAA Space Flight Mechanics Conference, Maui, Hawaii, February 8-12, 2004, pp. 1893-1904.