SSA positional and dimensional accuracy requirements for Space Traffic Coordination and Management

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ABSTRACT

Spacecraft operators employ diverse close approach metrics and standoff distances when determining whether a collision avoidance maneuver is warranted. Typically, operators with spacecraft in a low-risk orbital regime may implement ultra-conservative collision avoidance strategies at little fuel or operations cost, while operators with spacecraft operating in high-risk regimes are forced into economical collision avoidance strategies to avoid depleting their fuel budget and overtaxing their flight dynamics teams.

Unfortunately, while many collision avoidance maneuver Go/No-Go criteria exist, operators are often unable to obtain the SSA information and SSA accuracy necessary to populate the criteria that suit them best. Additionally, the algorithms used to populate these criteria sometimes contain invalid assumptions such as using linearized collision probability and spherical object shape approximations when more sophisticated formulations are required. And while some sources exist for estimated satellite object dimensions, the relative attitude at the time of conjunction may be uncertain or even unavailable, particularly for the so-called “secondary” or conjuncting object.

The Space Data Association (SDA), an association of global satellite operators working to ensure a controlled, reliable and efficient space environment, has run a survey among its Members to gather data on their Conjunction Assessment concept of operations. These include collision avoidance Go/No-Go metrics, collision avoidance targets, and operational constraints. Any entity attempting to provide a meaningful conjunction assessment service to operators could use these data to design the requirements over the service.

This paper assesses the various positional accuracy requirements of Space Situational Awareness (SSA) data associated with each of these diverse “Go/No-Go” metrics as employed in the conjunction mitigation processes used for Space Traffic Coordination (STC) and Spaced Traffic Management (STM). These metrics include miss distance at the Time of Closest Approach (TCA), componentized miss distance (e.g., TCA radial separation to preclude collision even when in-track or cross-track separations or uncertainties are unknown), and maximum collision probability and estimated true probability.

A further relationship to be explored is the dependence of collision probability on orientation and configuration/shape of the satellites at TCA. Lack of knowledge in orientation necessitates certain assumptions when computing collision probability. A common practice is to approximate a spacecraft’s hardbody with an encapsulating sphere. This one-shape-fits-all approach eliminates the need to determine orientation, but results in an overestimated object volume and an overinflated probability unless both satellites are actually spheres. To produce more representative probabilities, we use a satellite’s dimensions to define an encompassing rectangular box. This more accurately portrays the actual collision threat by projecting a smaller area than a sphere, the downside is that the box’s orientation to some level of accuracy must be known. But even when choosing the orientation that produces the maximum possible footprint, the probability of the box shape will be less than that of the sphere. To address this, we estimate a spectrum of collision probability values corresponding to a range of orientations, from which we can explore the interrelationship between attitudinal knowledge and position accuracy required for a given collision probability threshold.

We also explore the interrelationships between SSA positional accuracy, the operator-selected Go/No-Go metric and its threshold, timeliness, and resulting maneuver frequency. For example, the necessity to perform a collision avoidance maneuver adhere to a squared relationship on the adopted miss-distance threshold. The miss distance threshold adopted by the operator should, if done properly, be a function of the estimated accuracy of the primary and secondary objects as a function of time.

We conclude this paper by comparing the accuracy requirements derived for each metric above with estimates of positional accuracy observed in actual SSA data fusion experiments conducted this past year. In many situations, the accuracy of legacy and commercial SSA systems is insufficient to support the adopted Go/No-Go metric without using comprehensive data fusion techniques.

1. INTRODUCTION

In an ideal world, a satellite's configuration, orientation, and future position would be known with absolute certainty. If two satellites were predicted to touch, the probability of collision (Pc) would be one, else Pc would be zero. Lamentably, we do not live in such a world. Surveillance and tracking sensors are imperfect, as well as orbit propagation techniques. With imperfect knowledge, estimating the possibility of a collision requires making some simplifying assumptions. Typically, we use spheres in the absence of satellite configuration/orientation information, and we use covariance to estimate positional uncertainties resulting in an entire range of collision probabilities. In the absence of covariance, miss distance screening and/or maximum probability[1,2] is used.

Historically, space flight safety and related aspects have had to be accomplished in a data-limited environment. In the early days of space object tracking, the single SSN accuracy requirement was that the object be tracked with sufficient accuracy that "track custody" would be maintained. Over time, SSN tracking accuracies continued to improve - - but not necessarily commensurate with the increasing and diverse ways that the space community found to use and incorporate this information. The space operations community, while very appreciative of any/all available tracking data it can obtain, has had to accept the innate accuracies of the best-available positional data (TLEs, ephemerides that lack planned or historical maneuvers, lack of uncertainty information, unknown object sizes, shapes, orientations, masses, and materials) to operate spacecraft and avoid collisions.

While new government, civil and commercial initiatives are making great strides to address these shortcomings, the mindset of "making the best of the data we have" remains the primary approach. Yet as was recently demonstrated in a Space Traffic Coordination and Management (STCM) demonstration, positional accuracy of current SSA products often is insufficient to be used in the way that operators are using it. The mindset needs to evolve to consider (a) what metrics and thresholds are required to promote space flight safety and long-term sustainability of space operations; (b) what positional accuracies do such metrics and thresholds require in order to be actionable; and (c) what sensor laydowns, data fusion processes, data exchange, orbit determination methods, covariance realism, and orbit propagation techniques are required to ensure that the resulting positional accuracy requirements are amply met.

Accuracy requirements with respect to conjunction avoidance parameters have been presented in several papers[3,4] and nicely summarized by Sánchez-Ortiz and Krag[5]. Those works examine the number of false alerts per year as well as risk reduction through sensor improvements. This work differs from those in that we use the concept of maximum probability to determine minimum accuracy required at the Time of Closest Approach (TCA). Required accuracy at orbit determination epoch can then be deduced by backwards-propagating the covariance in an orbit-dependent manner.

2. COLLISION AVOIDANCE "GO/NO-GO" METRICS

When a spacecraft operator selects metrics and corresponding thresholds to help them judge when an upcoming close approach is too close for comfort, they likely incorporate not only their operational knowledge, but also the complexity and computational resources required to assess the metric, input data required by the metric, the amount of regime crowded, flight dynamics and management staffing, corporate and cultural considerations.

One might tend to think that operators have evolved to a consensus, standardized view of what go/no-go metric to use. Yet there are today numerous metrics, and even combinations of metrics, that operators employ. The diversity of these metrics often is driven by the vastly different environments, mission funding levels, resources, and collision risks that a particular spacecraft (or operator) tends to face.

Spacecraft operators in all regimes often struggle to determine which conjunctions are "too close." Operators with spacecraft operating in a low-risk orbital regime can implement simple yet effective, ultra-conservative collision avoidance strategies at little fuel or operations cost. Operators with spacecraft operating in high-risk regimes must be as realistic and as "lean" in their collision avoidance strategies as possible to avoid depleting their fuel budget and overtaxing their flight dynamics teams. Unfortunately, while many collision avoidance maneuver Go/No-Go criteria exist, operators are generally unable to obtain the metrics and data types necessary to populate the criteria that suit them best. Additionally, the algorithms used to populate these criteria sometimes contain invalid assumptions such as using linearized collision probability and spherical object shape approximations when more sophisticated formulations are required.

While collision probability (P_C) has become a popular criteria when assessing conjunction threats in some orbital regimes, its use is certainly not universal. Unlike other singular methods such as Cartesian distance, Mahalanobis distance, maximum probability, or ellipsoids-touching tests, many operators prefer P_C-based action thresholds because P_C incorporates miss distance, covariance size and orientation and the sizes of the conjuncting objects in a mathematically rigorous fashion. Additionally, collision probability metrics can be compared on an equal footing with other failure scenario probabilities such as the probability that a thruster would "stick open."

Yet widespread adoption of P_C by operators is ill-advised (and unlikely) for several reasons. First, the data required to assess P_C may either not be available, or not available at the accuracy required to obtain decision-quality P_C estimates. Second, operators may experience so few conjunctions (e.g., in MEO) that they have ample maneuvering fuel to take a more conservative approach such as the use of a miss distance threshold, greatly simplifying the analyses required of their flight dynamics staff.

With these thoughts in mind, consider the non-exhaustive list of go/no-go metrics operationally used by spacecraft operators for flight safety as provided in Table **1**. There is a spectrum of criteria being used, ranging from ultra-conservative maximum probability metrics that are mathematically rigorous and quite useful when there are few conjunctions and maneuvering fuel is ample, to purely miss distance-based screening using arbitrary thresholds, to estimated actual P_C.

The notional 'ratings" included in Table **1** were purely subjective, as judged by peers knowledgeable in the algorithms being used. At times, the rating is listed as a question mark "?", denoting that a rating is not possible without knowing what the user selected as a threshold. Where possible, the table has been sorted by the estimated amount of maneuvering fuel required, with a value of ten denoting the least use of fuel. Nevertheless, they reveal a few interesting traits:

1. Metrics that are based upon arbitrary miss distance criteria, while quite simple to evaluate, provide an unknown level of "protection" and can be very inaccurate in portraying actual collision risk.
2. The criteria that are the simplest to evaluate and require the least amount of input data tend to require the greatest maneuvering fuel (and the greatest number of avoidance maneuvers).
3. The criteria that use the least amount of maneuvering fuel tend to be judged the most accurate in terms of quantifying the likelihood that a collision would occur.

Table 1: List of operationally used Go/No-Go conjunction screening criteria, characterized on a scale of one to ten

Collision Avoidance Go/No-Go Criterion	Conservatism (10=most conservative)	Fuel Usage (10 = least fuel)	Accuracy (10 = best risk portrayal)	Complexity and data required (10=simple, little data)
Cartesian miss distance, arbitrary user threshold)	*	*	1	10
Componentized miss distance, arbitrary user threshold (e.g., radial-only separation)	*	*	1	10
Combination of miss distance and estimated collision probability P_C (e.g., "F-Factor" [6])	*	*	7	4
Max Probability-based Cartesian miss distance	10	2	3	9
Eigenvalue-based componentized miss distance	8	4	4	4
Collision "Consequence" metric [7]	5	5	6	3
Mahalanobis miss distance	9	5	3	4
Mahalanobis distance adjusted for spherical shapes (combined hard-body radius or **CHBR**)	9	5	5	4
P_C, linearized motion, spherical CHBR	8	7	7	4
P_C , non-linear motion, spherical CHBR	3	7	7	3
P_C , linearized motion, asymmetric body shapes	6	9	9	2
P_C , non-linear motion, asymmetric body shapes	1	10	10	1

* Dependent upon selected threshold(s)

Table **1** indicates that from a fuel usage standpoint, the most accurate form of collision probability assessment would serve as the best conjunction assessment metric. But collision probability assessment has its shortcomings as well. P_c should not be used as a Go/No-go metric without first fully understanding the potential inaccuracies, assumptions and pitfalls associated with it. Many of these are discussed in [[8]]. Of principal concern are:

(1) Nominal trajectories may be inaccurate, primarily due to unforeseen (and therefore unmodelled) maneuver(s) on the part of either your satellite, or the object you're conjuncting with, but also due to other unmodelled forces and perturbations (e.g., space weather event or atypical attitude orientation or attitude maneuver etc.);
(2) Covariance (error) information may be inaccurate or unavailable for either your satellite or the object you're conjuncting with;
(3) The conjunction may be "non-linear," violating the assumptions of the simpler P_c assessment methods;
(4) The object shapes may be aspherical, violating the hard body radius assumptions of the simpler P_c assessment methods;
(5) The hardbody size of your satellite might not be properly reflected in the assessment system;
(6) The hardbody size of the object you're conjuncting with might not be known and/or properly reflected in the assessment system.

Each of these six concerns can lead to P_c estimates that are multiple orders-of-magnitude from the actual P_c estimates one would obtain if none of these principal concerns existed.

Once the operator has selected the metric that they want to use to assess how concerning an upcoming close approach is, the operator must select the threshold (or as we will see in the next section, the combination of thresholds) that serve as the trigger for when to conduct a collision avoidance maneuver. In selecting the threshold(s) of concern, a spacecraft operator might consider such diverse aspects as:

- Importance of the mission (critical to human health/safety, military, communications, earth imaging, or merely educational, etc.);
- How long it may take to field a replacement spacecraft, should their current one be destroyed or impaired by the collision;
- How well-staffed an operator's flight dynamics team is to be able to process and avoid conjunctions.
- Frequency of close approaches (for example, if an operator's spacecraft rarely comes close to other spacecraft or debris, then that operator can afford to be quite conservative in their approach to guarantee that their spacecraft is safe without adversely impacting mission duration and depleting fuel prematurely)
- Public awareness and/or opinion;
- Investment, level of interest, and involvement of their shareholders;
- Cost of the spacecraft;
- Cultural aspects;
- Concerns over competitive 'shaming';
- Competitive advantage.

It is easy to see from this diverse list how operators may employ diverse metrics and thresholds. As an aside, note that such considerations as the long-term sustainability of space activities, while certainly very important to many commercial companies, may not be a top consideration for some companies when selecting their metrics and thresholds. And the thresholds that the operators select may primarily be targeted at ensuring the safety, security, and availability of their individual spacecraft and mission services, as opposed to ensuring global space safety.

3. SDA OPERATORS SURVEY

With those things in mind, it is useful to "take the pulse" of the spacecraft operator community to see what Go/No-Go metrics and thresholds are actively being used today. One approach to obtaining such information is by surveying operators that participate in an industry-formed association.

The Space Data Association (SDA) is an association of satellite operators which has the primary goal of mitigating the risk of proximity operations and facilitating operational coordination among its members. The SDA comprises 32 operators in all orbital regimes (LEO, MEO, and GEO). These operators agreed to pool their operational data to perform ephemeris-vs-ephemeris conjunction assessment using best accuracy data. The Space Data Center currently performs flight safety assessments for 274 GEO satellites (over half of all active satellites in GEO). Additionally, 475 LEO/MEO satellites are handled by the SDA, which performs conjunction assessment runs several times a day.

The SDA supported the US Department of Commerce (DoC) in the design of a Pilot Project in the field of Space Surveillance and Tracking. This Pilot Project was expected to be preparatory to the development of a full-scale service that could take over the Conjunction Assessment service role currently performed by the 18^{th} Space Control Squadron (18^{th} SPCS).

In the framework of this activity, the SDA has collected anonymized information about the Collision Avoidance Concept of Operations (CA ConOps) of its members. The idea was to understand what performances a new system should deliver to help operators in their everyday job of Conjunction Assessment and Collision Avoidance.

Collision Avoidance operations are always platform-dependent, so each operator effectively has a different CA ConOps for each type of spacecraft. For this reason, the SDA also collected anonymized data on the maneuver capabilities of Members' spacecraft.

The SDA gathered voluntary feedback from 13 GEO operators (200 satellites) and 7 LEO operators (394 satellites), where LEO is defined as satellites orbiting at an altitude comprised between 400 and 2000 km. The main purpose of the survey was to understand what criteria the operators use when deciding whether or not to perform a collision avoidance maneuver, what are the main challenges in executing such operation, and what is the outcome that the operator is trying to achieve. In what follows, the main results will be presented and commented.

3.1 High-Interest Close Approaches

Question: *Which parameters do you monitor and which thresholds do you use to decide whether a conjunction event is of high interest?*
This question is asking operators to elaborate on when a conjunction warning prompts further analysis from the operations team. This does not necessarily translate into executing a collision avoidance maneuver, as there is the possibility to reshuffle station-keeping maneuvers in such a way to change the geometry of the encounter.

3.1.1 GEO Results

Some operators monitor exclusively the miss-distance, analyzing any event that results in an object entering a spherical volume centered on the spacecraft. These operators usually use 10km as the sphere diameter, the default value for deep-space mission screening used by the 18^{th} SPCS.

Most operators use a combination of different parameters. The most common parameter monitored together with the tridimensional miss-distance is actually the radial component of the miss-distance. This is due to the fact that this component is usually the one with the lowest uncertainty. Only a few operators among those contacted have decision criteria that consider also the along-track and cross-track components of the miss-distance.

Approximately half the operators contacted monitor statistical parameters together with geometrical parameters. The probability of collision is not included in GEO CDMs, but the SDA provides the maximum probability of collision[1], and some operators make their own estimates of the probability of collision by introducing assumptions on the dimensions of the secondary object.

The actual values of the monitored parameters that trigger higher conjunction warning scrutiny differ from operator to operator, and depend on their concept of operations, staffing constraints, satellites capabilities, and orbit determination performances. The results for GEO are summarized in Figure 1 and Table 2.

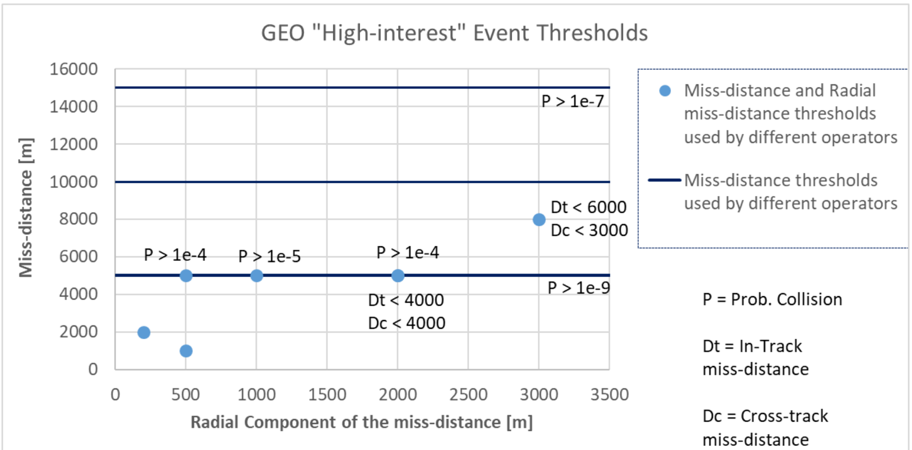

Figure 1: GEO "High Interest" Event Thresholds

Table 2: GEO "High Interest" Event Thresholds (each column represents an operator)

	GEO 1	GEO 2	GEO 3	GEO 4	GEO 5	GEO 6	GEO 7	GEO 8	GEO 9	GEO 10	GEO 11	GEO 12	GEO 13
Probability of Collision ≥ P		1E-04	1E-07	1E-05		1E-04						1E-09	
Max Probability of Collision (Alfano Method) ≥ P		1E-04		1E-05									
Tri-dimensional miss distance < D [m]		5000	15000	5000	10000	5000	8000	2000	5000	10000	1000	5000	
In-Track component of the miss distance < Dt							6000		4000				
Cross-Track component of the miss distance < Dc							3000		4000				
Radial component of the miss distance < Dr		2000		1000		500	3000	200	2000		500		
Combination of two or more of the above parameters	YES	YES	YES	YES		YES	YES	YES	YES		YES	YES	

- The parameters monitored and thresholds used vary greatly among operators. The operators who responded to this question use these values:
- Tridimensional miss distance in the range [1 km – 15 km], with a median of 5 km;
- Radial component of the miss distance in the range [0.2 km – 3 km], with a median of 1 km;
- Probability of collision in the range [10^{-9} – 10^{-4}], with median of 10^{-5}.

3.1.2 LEO Results

There is more consensus among LEO operators regarding the parameters to monitor and the values used to trigger further analysis. All the operators that responded monitor the probability of collision. This is because LEO CDMs

already provide these estimates. A majority of responders also monitor the tridimensional miss-distance. None of the operators reported monitoring any other parameter other than these two.
Most LEO operators use values in the range [100 m – 1000 m] for the tridimensional miss-distance, and [10^{-5} – 10^{-4}] for the probability of collision.

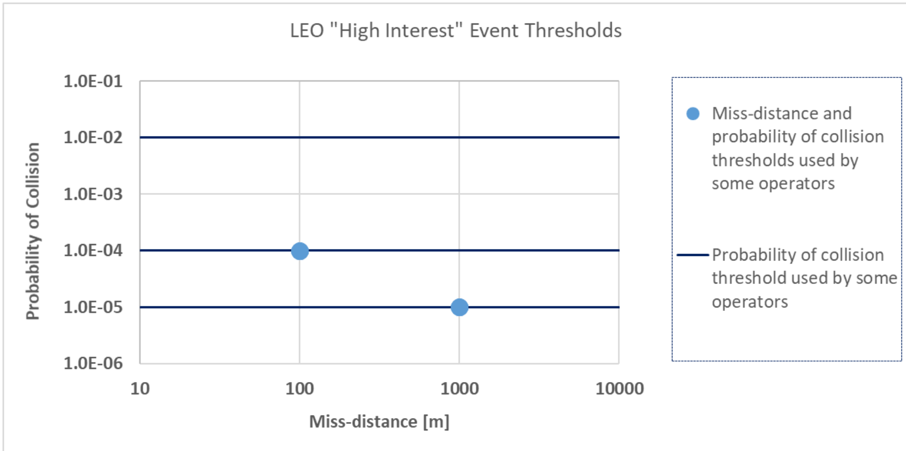

Figure 2: LEO "High Interest" Event Thresholds

Operators whose fleet is non-maneuverable represent an exception. The only mean these operators have of mitigating the risk of a close approach is by executing an attitude maneuver and relying on differential drag to modify the satellite orbit. These operators usually need to interrupt the mission of their satellites for the duration of these maneuvers, which can last several days. For this reason, these operators tend to use higher thresholds for the probability of collision – reportedly, 10^{-2}.

3.2 Collision Avoidance Maneuver Execution Time

Question: *If you have ever executed a (chemical, electric, or else) maneuver specifically designed for Collision Avoidance, what is your preferred maneuver execution time with respect to the Time of Close Approach (TCA)?*

This question is important to assess operators' decision loop timing for Collision Avoidance. There are two schools of thought. Some operators act as early as possible to mitigate the risk of close approach well before this risk results in a high probability of collision. Other operators wait as long as possible to refine the available data and get a better estimate of the geometry of the encounter and its risk level. The spacecraft's propulsion system clearly affects the concept of operations, as the next section will show.

3.2.1 GEO Results

The preferred execution time for a Collision Avoidance maneuver, as reported by GEO operators, is summarized in Figure **3**,

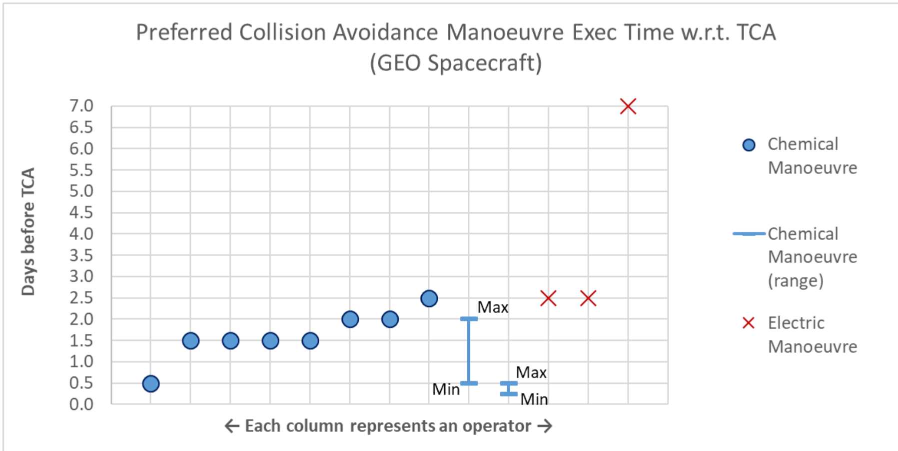

Figure 3: GEO Preferred Collision Avoidance Maneuver Execution Time w.r.t. TCA

As Figure **3** shows, the preferred execution time for a Collision Avoidance maneuver in GEO is usually between 0.5 and 2.5 days before TCA, with a median value at 1.5 days. This is because the collision avoidance maneuver is usually an in-plane maneuver to increase the radial component of the miss-distance. Only one operator explicitly declared also using out-of-plane maneuvers for collision avoidance purposes – in this case the execution time was 6 hours before TCA in order to maximize the variation of the cross-track component of the miss distance. One electric propulsion spacecraft operator reported that collision avoidance is carried out seven days before TCA. While this value is an outlier among those reported, it is worth remembering that most electric propulsion spacecraft plan the station-keeping cycle for a whole week, so this can introduce planning constraints.

3.2.2 LEO Results

A majority of the LEO operators contacted prefer to execute a Collision Avoidance maneuver about 72 hours before TCA, independently of the propulsion system used, as Figure **4** summarizes. One operator indicated the preference to perform chemical maneuvers around 6 hours before TCA, while another scheduled electrical maneuvers about 24 hours before TCA.

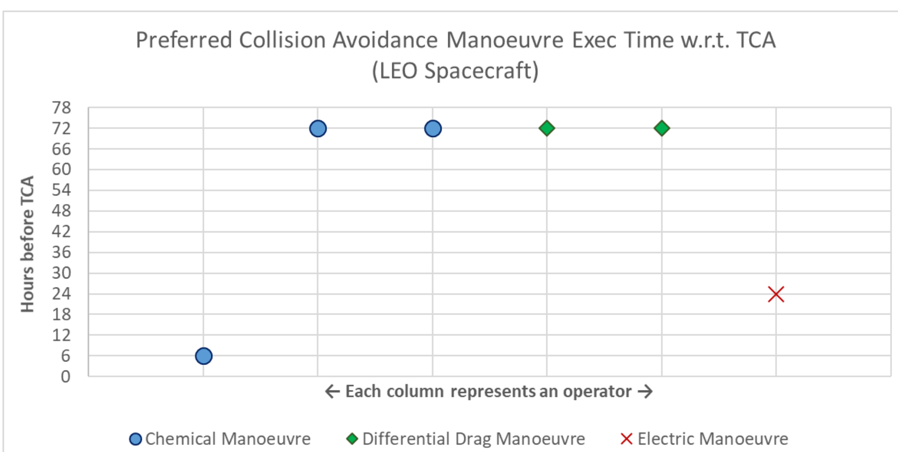

Figure 4: LEO Preferred Collision Avoidance Maneuver Execution Time w.r.t. TCA

Furthermore, one more operator simply indicated the preference to perform collision avoidance less than one orbital period before TCA (this data point is not shown in Figure **4**).

3.3 Collision Avoidance Maneuver Preparation

Question: *How many hours can pass between notification of an emergency and the execution of the collision avoidance maneuver?*

This question essentially aims at estimating how long the reaction time can be after the notification of an emergency.

3.3.1 GEO Results

The responses on reaction time provided by GEO operators are summarized in Figure **5**.

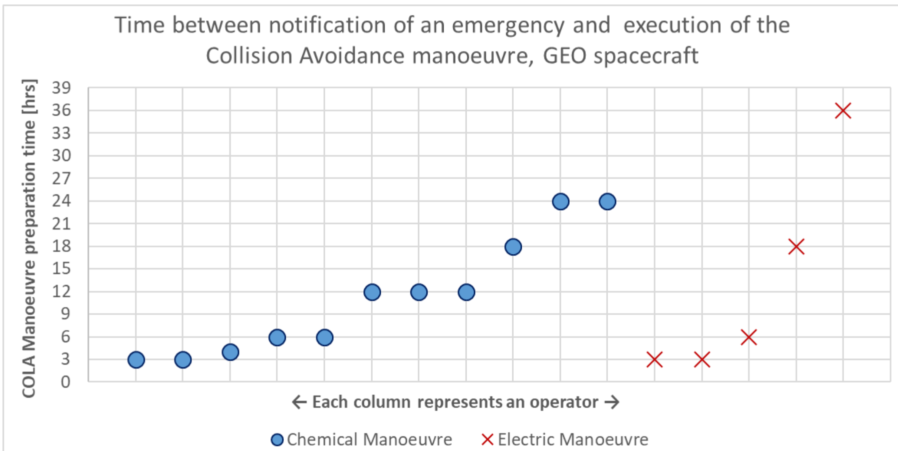

Figure 5: GEO Reaction Time

As Figure **5** shows, reaction time varies greatly among operators. This is likely depending on the concept of operations used. If the team assessing conjunctions and planning collision avoidance maneuvers works on 24/7 on-call shifts, then the reaction time can be quite short. If the team works on nominal working hours, then the reaction time can increase significantly, especially if the notification of the emergency is delivered during the weekend. All operators reported values between 3 and 36 hours, and a majority reported a value of 12 hours or less. There is no significant difference between the reaction time for chemical or electric propulsion.

3.3.2 LEO Results

All LEO operators who chose to answer this question reported response times equal to or lower than 12 hours, without significant difference between the reaction time for chemical or differential drag maneuvers.

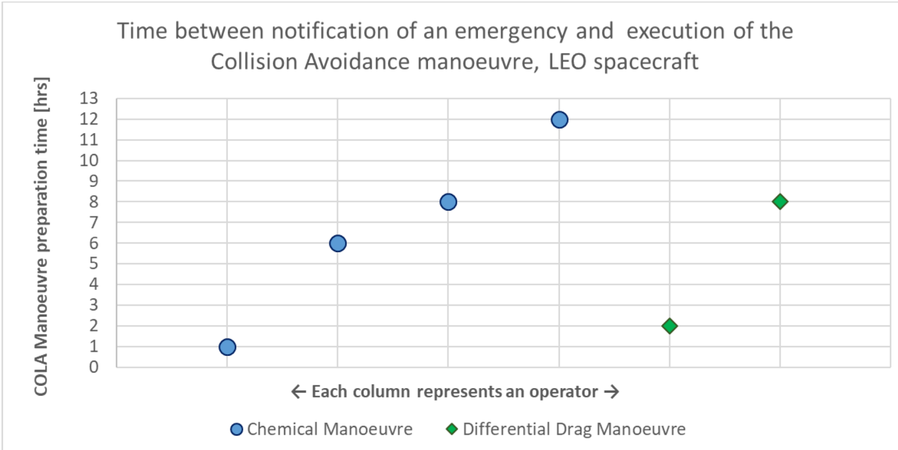

Figure 6: LEO Reaction Time

3.4 Collision Avoidance Maneuver Targets

Question: *If you have ever executed a (chemical, electric, or other) maneuver specifically designed for Collision Avoidance, what parameter do you try to alter and which value do you try to obtain?*

This question is essentially asking the operators to elaborate on when they would consider a high-interest event successfully de-risked. Mitigation is usually the result of reshuffling of planned maneuvers, deletion of maneuvers already commanded to the spacecraft, or planning a dedicated collision avoidance maneuver. To assess how the conjunction risk would evolve after implementing any of the aforementioned changes, operators would request updated estimates from the relevant conjunction assessment service.

3.4.1 GEO Results

In section 3.1.1, we mentioned that there is no clear consensus among GEO operators on which condition defines a high-interest event. Similarly, there is no wide consensus on which parameters to target to mitigate the risk of close approach, as Figure **7** and Table 3 shows.

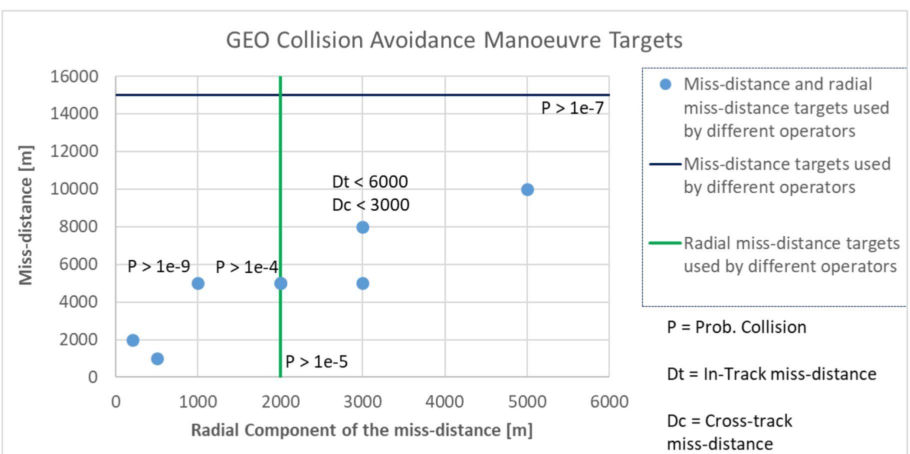

Figure 7: GEO Collision Avoidance Maneuver Targets

Table 3: GEO Collision Avoidance Maneuver Targets (each column represents an operator)

	GEO 1	GEO 2	GEO 3	GEO 4	GEO 5	GEO 6	GEO 7	GEO 8	GEO 9	GEO 10	GEO 11	GEO 12	GEO 13
Probability of Collision > P		1E-04	1E-07	1E-05								1E-09	
Max Probability of Collision (SDA emails) > P		1E-04		1E-05									
Miss distance > D [m]		5000	15000		5000		8000	2000	5000	10000	1000	5000	
In-Track component of the miss distance > Dt							6000						
Cross-Track component of the miss distance > Dc							3000						
Radial component of the miss distance > Dr		2000		2000	3000		3000	200	2000	5000	500	1000	
Combination of two or more of the above parameters		YES	YES	YES	YES		YES	YES	YES	YES	YES	YES	

The one thing that all GEO operators have in common is that they target at least one geometric condition (i.e., a certain value of the miss-distance and/or of one or more of its components). About half of the responding operators target both geometric and statistical conditions, i.e. they target a certain geometry and a maximum probability of collision. In summary, the operators contacted use these target values:

- Tridimensional miss distance in the range [1 km – 15 km], with a median at 5 km;
- Radial component of the miss distance in the range [0.2 km – 5 km], with a median at 2 km;
- Probability of collision in the range [10^{-9} – 10^{-4}].

3.4.2 LEO Results

The collision avoidance maneuver targets in LEO are fairly consistent among the responding operators. All of them target a certain probability of collision, while some of them also target a minimum miss-distance, as Figure **8** summarizes.

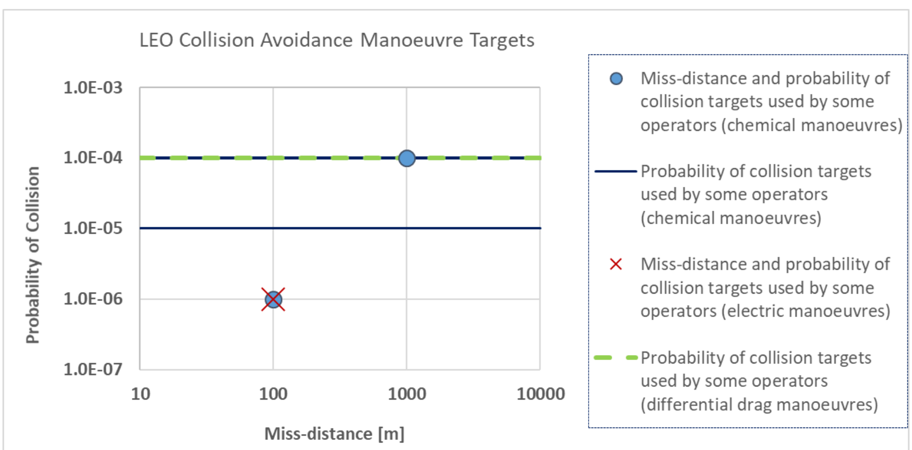

Figure 8: LEO Collision Avoidance Maneuver Targets

All the operators target probability of collision between 10^{-6} and 10^{-4}. Half of them target a minimum miss-distance, with values in the range [100 m – 1000 m]. There is no evidence to suggest that operators who execute a specific kind of collision avoidance maneuver (chemical, electrical, or differential drag) target values much different from those used by other operators.

4. DEPENDENCE OF COLLISION PROBABILITY ON OBJECT SHAPE AND ORIENTATION

When performing conjunction analysis for short-term encounters, it is often the case that the orientation and configuration/shape of the satellites are unknown. As was demonstrated in [9 10], the shape, size and dimensions of each of the two conjuncting space objects plays a critical role in the estimation of collision probability for a conjunction event. It is almost exclusively the case for debris objects. This necessitates certain assumptions when computing collision probability. A common practice is to approximate a spacecraft's hardbody with an encapsulating sphere. This one-shape-fits-all approach eliminates the need to determine orientation, but results in an overestimated object volume and an overinflated probability unless both satellites are actually spherical.

To produce more representative probabilities, Figure 9 shows an enveloping rectangular box about a satellite of length (*l*), width (*w*), height (*h*) of 13m, 4.3m, and 1.6m respectively. This representation more accurately portrays the actual collision threat by projecting a smaller area on to the conjunction encounter plane than a sphere[11], the downside is that the box's orientation must be known. Not knowing the orientation, we use uniformly spaced viewing angles (Figure 10) to provide a spectrum of values in ascending order for all projections (Figure 11). The user then has the freedom to choose a suitable range of orientations. As was shown in our previous work [10], even when choosing the maximum footprint possible, the resulting probability of the box will be less than that of the sphere.

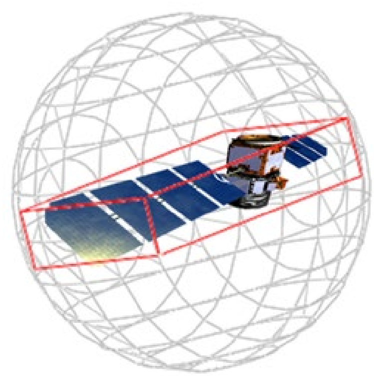

Figure 9. Sphere of minimal volume touching all corners of a rectangular box enveloping the satellite

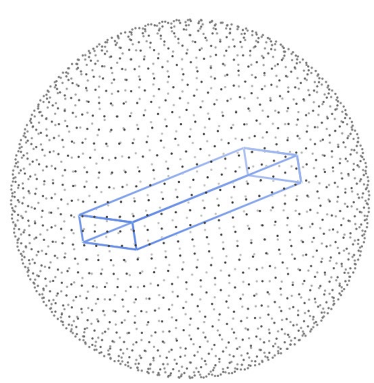

Figure 10. Box surrounded by equally-spaced viewing points

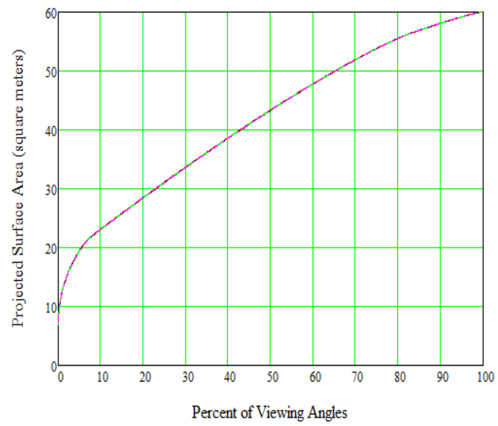

Figure 11. Satellite area projections in ascending order

Figure 11 reveals that 80% of the viewing angles will observe a surface area at or below $56m^2$, 50% below $44m^2$, and so on. The associated radii for representative circles in the encounter plane are computed using a method similar to Chan's Method of Equivalent Cross-Sectional Area (MECSA)[12]. Unlike Xie and Chan's approach, the rectangular dimensions and orientation are redefined in the encounter plane rather than converting to a circle, thus simplifying the integrable region. The resulting radius distribution is shown in Figure 12. From Figure 10 and Figure 11, we see that the box's largest projected area is $60m^2$ which will produce a circle of equivalent area with radius of 4.37m.

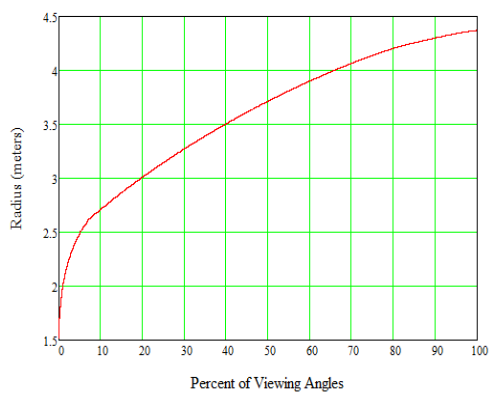

Figure 12. Radii of satellite's area projections

The sphere's diameter is 6.89m with a projected area of 149.3 m^2 regardless of viewing angle. In the encounter plane, such a circle will envelop the largest possible box plus an additional 89.3 m^2 of density space. Thus, for the same centroid, the box's smaller footprint will produce a lower and more reasonable probability. This holds true for all cases because the encapsulating circle will always contain more probability density space than a box's projected maximum area.

The above process is applied to each conjuncting satellite's dimensions to produce its projected areas/radii. The minimum, maximum, and/or user-choice percentages of the box are used to establish their respective radii. When modeling an encapsulating sphere, its radius is used instead. Summing the radii for both objects determines the combined hardbody radii (CHBRs). In addition to representation as a circle, Reference 10 also describes and demonstrates the use of squares and rectangles; its latter case is given below as an example.

US data predicted a close approach between the Infrared Astronomical Satellite (IRAS, NORAD ID 13777) satellite and the Gravity Gradient Stabilization Experiment (GGSE-4, NORAD ID 02828), forecast to occur on January 29, 2020, 23:39 GMT at roughly 900 km altitude. Both satellites were inoperable and therefore incapable of maneuvering. Progressive conjunction information from the 18th SCS repeatedly showed a miss distance under 20m. Fortunately, the collision did not occur.

IRAS's box dimensions [3.6m, 3.6m, 2.05m] and GGSE-4's dimensions [18m, 0.7m, 0.6m] were used along with orbital data to produce the following figures. GGSE-4's encapsulating sphere was quite large due to its long protruding boom. This made it a good candidate to compare and contrast with the box's equivalent area representations.

Using each object's maximum and minimum projections, Figure 13 and Figure 14 show that the satellites' equivalent projected areas produce considerably lower probabilities than encapsulation by eliminating density space. The values for the projected square and MECSA circle are so close that the lines somewhat overlap. GGSE-4's elongated shape causes a large difference between maximum and minimum projected areas, resulting in a large range of associated Pc values. More details can be found in Reference 10.

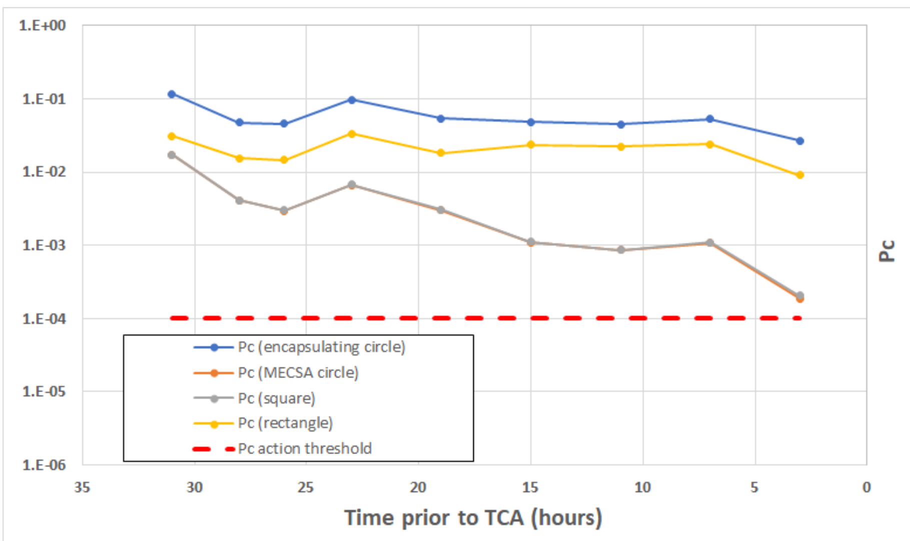

Figure 13. IRAS/GGSE-4 maximum projections.

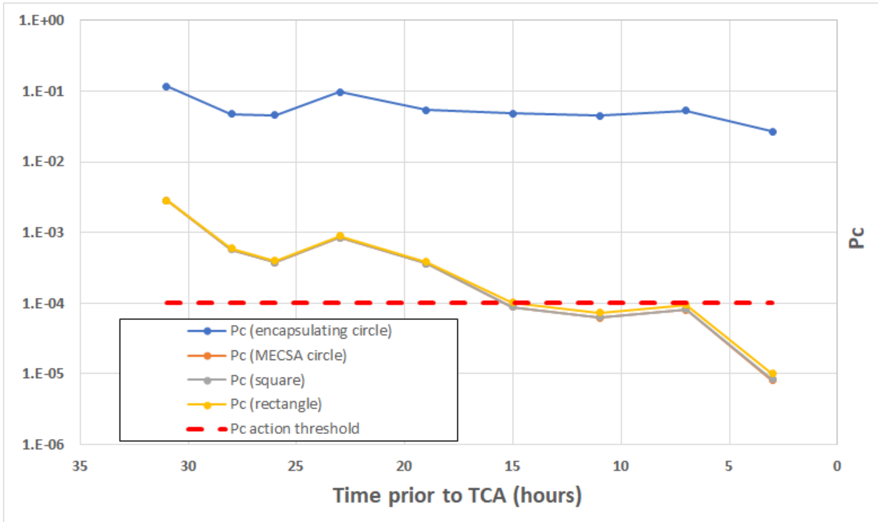

Figure 14. IRAS/GGSE-4 minimum projections.

5. POSITIONAL ACCURACY

When using probability as a decision metric, the relationship between miss distance $\boldsymbol{d}$ and absolute maximum probability $\boldsymbol{P_{max}}$ for spherical objects can be approximated if the combined object radius $\boldsymbol{r}$ and covariance aspect ratio $\boldsymbol{AR}$ are known [13]. $\boldsymbol{AR}$ is the ratio of the covariance major axis to its minor axis in the encounter plane. For this analysis, $\boldsymbol{P_{max}}$ occurs when the combined object's center lies on the major axis. The relational equation for linear relative motion is reasonably approximated by the analytical expression

$$P_{max} \cong \left(\frac{\alpha}{1+\alpha}\right)\left(\frac{1}{1+\alpha}\right)^{\frac{1}{\alpha}} \tag{1}$$

where α is

$$\alpha = \frac{r^2 AR}{d^2} \qquad (AR \geq 1) \tag{2}$$

Figure 15 shows the corresponding nomogram.

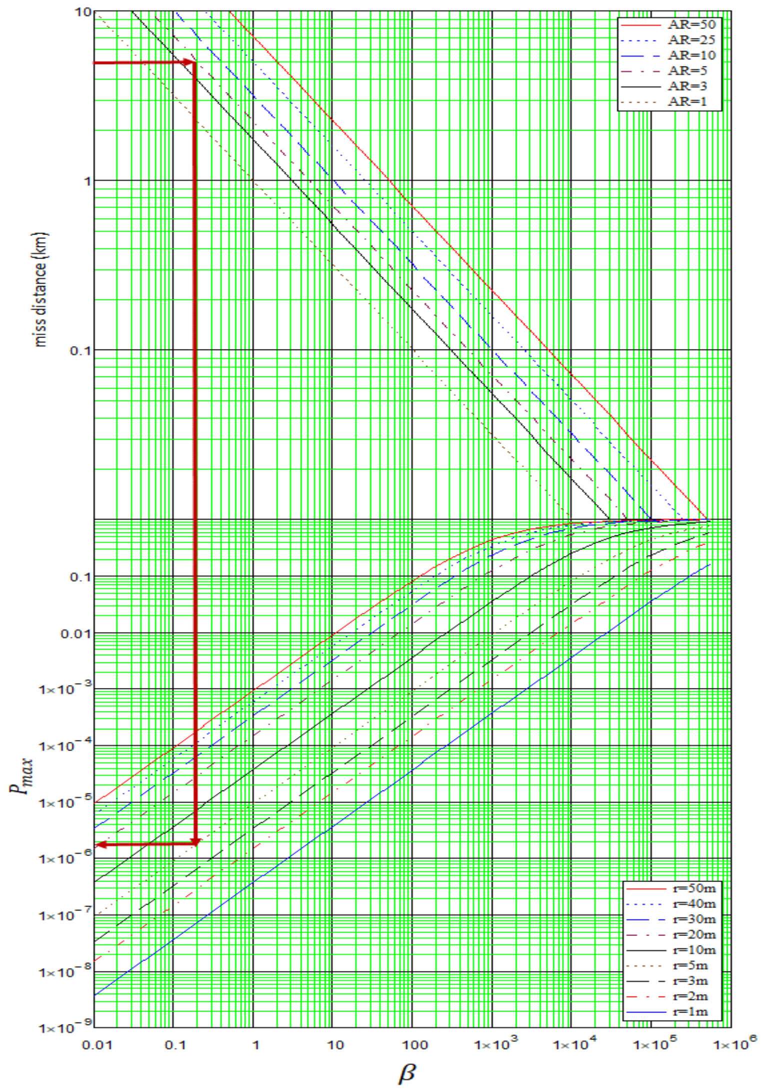

Figure 15. Example of using linked nomograms to determine P_{max} from r, d, and AR.

The major-axis standard deviation σ_{major} associated with P_{max} at TCA is found through the equation

$$\sigma_{major}(TCA) = \sqrt{\frac{-\eta}{2 \cdot ln\left(\frac{d^2}{d^2+\eta}\right)}} \quad (AR \geq 1) \tag{3}$$

where η is

$$\eta = AR \cdot r^2 \ . \tag{4}$$

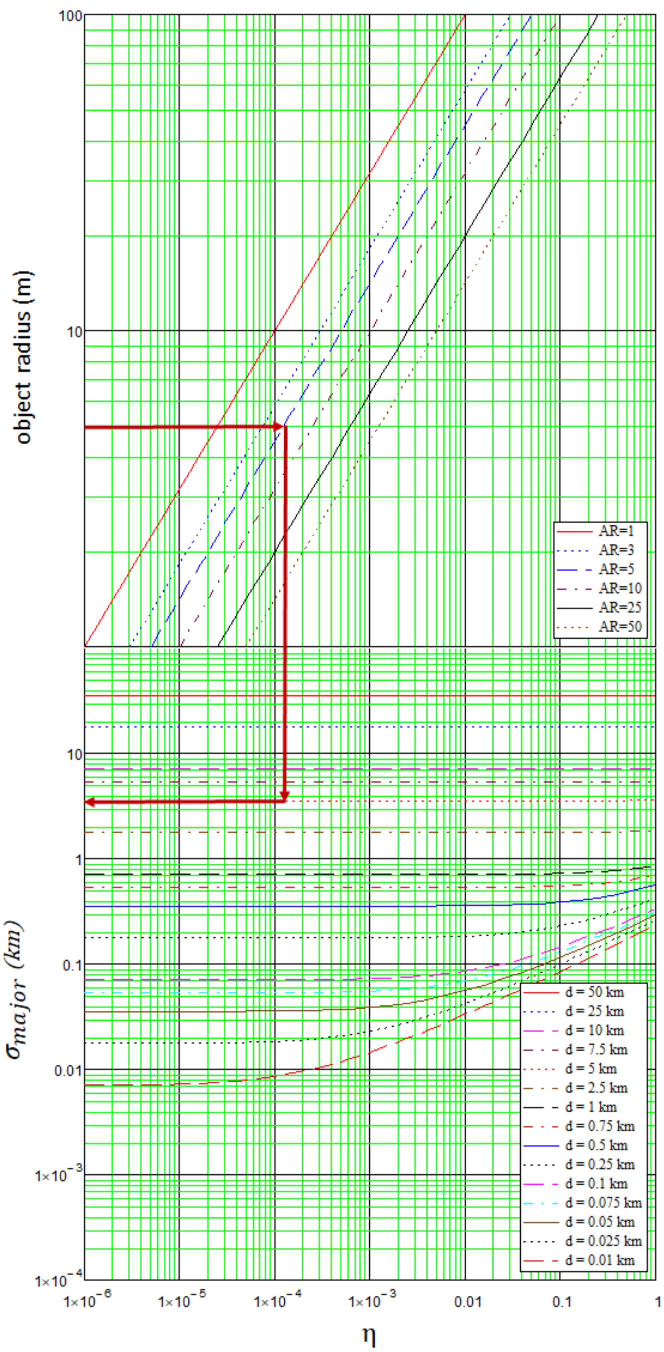

Figure 16. Nomogram to determine σ_{major} from r, d, and AR.

A combined object radius r of 5 meters, coupled with a covariance aspect ratio AR of 5 and a miss distance d of 5 kilometers, results in a P_{max} value of $1.84\text{x}10^{-6}$. As shown in the Figure 16 nomogram, the corresponding major-axis combined standard deviation σ_{major} is 3.5 km; its actual computed value is 3.535538 km. This standard deviation is associated with TCA and not orbit epoch. Although Equation 1 is independent of orbit regime, covariance propagation from TCA is not; such dependence will affect the covariance at epoch. Knowing that orbit propagation causes the covariance to grow, it is necessary to work backwards to orbit epoch to determine the appropriate accuracy requirement that ensures the probability calculation does not occur in the dilution region. As the nomogram reveals, accuracy cannot be reduced to a single number; it is dependent on the combined object radius, covariance aspect ratio, and miss distance at TCA.

The following table provides required accuracies associated with common conjunction screening values, produced using the exact relationships and equations. Note that for a typical Low Earth Orbit (LEO) Combined Hard Body Radius (CHBR) of 1 meter and an operator's P_c threshold of five in ten thousand (P_{max} = 5.E-04), the individual allowable major eigenvalue's corresponding one-sigma accuracy should be no greater than 24 meters (highlighted). In Geosynchronous Earth Orbit (GEO), a typically larger spacecraft (CHBR = 5m) might yield an allowable one-sigma accuracy of no greater than 117 meters (highlighted). These are very demanding requirements.

Table 4. Maximum allowable one-sigma error ellipsoid dispersion for assorted combinations of maximum probability and CHBR for aspect ratio AR=3.

P_{max}	CHBR (m)	AR	distance (m)	combined σ_{major} (m)	individual σ_{major} (m)
1.E-04	0.5	3	53	37	26
1.E-04	1	3	105	74	53
1.E-04	1.5	3	158	111	79
1.E-04	5	3	525	371	263
1.E-04	10	3	1050	743	525
1.E-04	20	3	2101	1486	1051
1.E-04	50	3	5252	3714	2624
5.E-04	0.5	3	24	17	12
5.E-04	1	3	47	33	24
5.E-04	1.5	3	70	50	35
5.E-04	5	3	235	166	117
5.E-04	10	3	470	332	235
5.E-04	20	3	939	665	470
5.E-04	50	3	2348	1661	1174
1.E-03	0.5	3	17	12	8
1.E-03	1	3	33	24	17
1.E-03	1.5	3	50	35	25
1.E-03	5	3	166	117	83
1.E-03	10	3	332	235	166
1.E-03	20	3	664	470	332
1.E-03	50	3	1659	1174	830

From Figure 16 it is apparent that σ_{major} is heavily dependent on d for most cases, almost to the exclusion of r or AR. Realizing this, a zero-order approximation $\tilde{\sigma}_{major}$ is simply

$$\tilde{\sigma}_{major}(TCA) = \frac{d}{\sqrt{2}} \quad (r \ll d\,,\ AR < 50). \tag{5}$$

The approximate value $\tilde{\sigma}_{major}$ becomes 3.535534 km, closely matching the previous value. Attributing equal uncertainty to both primary and secondary objects yields half the miss distance at TCA.

$$\sigma_{primary}(TCA) = \sigma_{secondary}(TCA) \cong \frac{d}{2} \ . \tag{6}$$

To assess componentized miss distance, the encounter plane's covariance ellipse is constructed such that the combined object contains all the probability mass associated with its minor axis[1]. This reduces the problem to a single-dimension analysis along the major axis. The componentized maximum probability P_{max_1d} is

$$P_{max\_1d} = \frac{1}{2}\left(erf\left[\frac{1}{2}\left(\frac{r}{d}+1\right)\frac{\sqrt{2}}{\sigma_{1d}(TCA)}\right] + erf\left[\frac{1}{2}\left(\frac{r}{d}-1\right)\frac{\sqrt{2}}{\sigma_{1d}(TCA)}\right]\right) \tag{7}$$

with an associated component axis standard deviation of

$$\sigma_{1d}(TCA) = \sqrt{\frac{2rd}{ln\left(\frac{d+r}{d-r}\right)}} \tag{8}$$

where d is the miss distance along the component axis.

6. MAHALANOBIS DISTANCE SCREENING

A bridge linking Cartesian and Mahalanobis spaces is found from the combined, positional, 3x3 covariance matrix C and relative position $[x \quad y \quad z]$. Mahalanobis distance d_{maha} is determined from the equation

$$d_{maha}{}^2 = [x \quad y \quad z] \ C^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{9}$$

where the three positional components represent the vector from the combined covariance center to the combined object's center at TCA. If one wishes to consider r, the vector components can be adjusted to touch the combined object's sphere closest to covariance center in the Mahalanobis space. This can be approximated by reducing the vector's magnitude by r while maintaining its directionality.

Decision criteria is based on the object being inside ($d_{maha} < n$) or outside ($d_{maha} > n$) the covariance ellipsoid's n-σ shell. Similarly, this approach can be dimensionally reduced to assess a single component for radial screening or dual-component, planar screening. Table 5 shows the probability density percentages contained within n-σ for various dimensions.

Table 5. Probability density percentages versus n-σ.

Dimension	1σ	2σ	3σ	4σ	5σ	6σ
1D	68.2689492%	95.4499736%	99.7300204%	99.9936658%	99.9999427%	99.9999998%
2D	39.3469340%	86.4664717%	98.8891016%	99.9664560%	99.9996274%	99.9999985%
3D	19.8748043%	73.8535870%	97.0709120%	99.8866067%	99.9984561%	99.9999925%

7. MANEUVER FREQUENCY DEPENDENCIES

7.1 Distance-based criteria

When used appropriately, the action threshold for a miss distance-based screening criterion should conservatively encompass the combined positional knowledge accuracy for the two conjuncting objects at the time of closest approach. This approach was listed as "Max Probability-based Cartesian miss distance" in Table 1. As has been demonstrated in many encounter rate characterization papers [9, 14, 15, 16], the inverse relationship between encounter frequency and the spherical radius for a miss distance metric indicated by kinetic gas theory's "time between collisions" is a very good approximation of the number of times that a miss distance threshold is violated in a given time span when the background space population is fairly homogenous. Under these assumptions, we can approximate how the number of avoidance maneuvers might scale from a baseline encounter rate for a 100m keep out sphere as a function of combined positional accuracy as shown in Figure 17. The highlighted example corresponding to a combined positional accuracy of 7 km indicates that the operator would have to do five thousand times more maneuvers than an operator who had highly accurate data whose combined positional accuracy was one hundred meters.

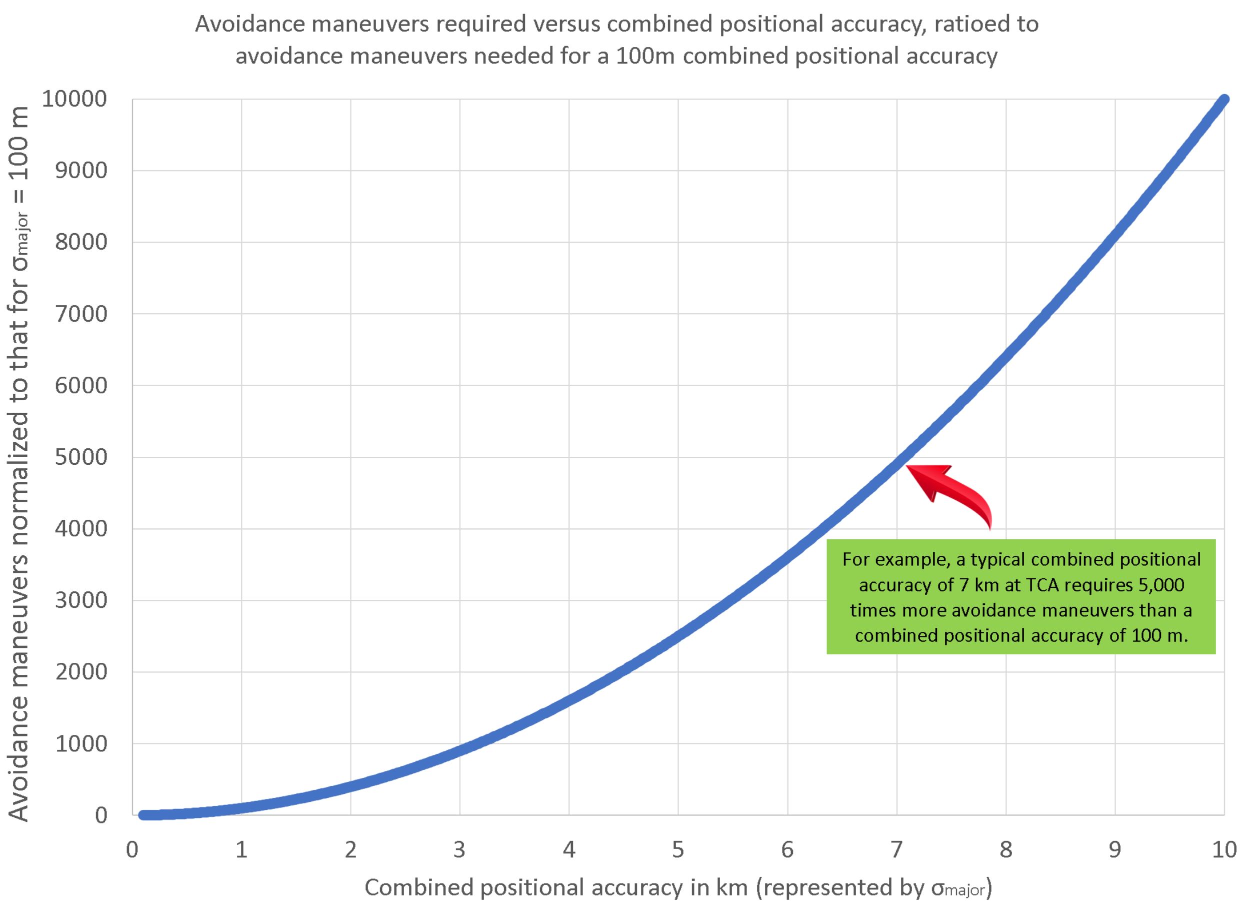

Figure 17. Encounter rate increase as a function of combined positional accuracy for distance-based screening.

The squared relationship on three-dimensional miss distance screening previously presented is based on the projection of a circle through the miss distance sphere normal to the encounter plane. The relationship works because for any encounter geometry, there is a circular encounter area being swept out in the relative velocity direction (i.e., normal to the encounter plane).

7.2 Componentized miss distance-based criteria

As presented in Section 2, operators may not trust certain components of the predicted miss distance vector at the Time of Closest Approach (TCA). Such mistrust may arise if the dominant propagation errors in the orbit regime in question lead to errors in certain component(s), or if the type of SSA sensor being used to gather the orbit tracking observations has a known weakness that can lead to large errors in one or more error component(s) when propagated forward. For example, operators have been known to emphasize the radial miss distance component and ignore in-track in a high-drag environment.

If the selected miss distance component direction lies in (i.e., is parallel to) the encounter plane for typical encounters, then one may expect the encounter rate to vary linearly as this miss distance threshold is increased, because the screening process will admit more conjunctions proportional to that component. But if the chosen miss distance component tends to run normal to the typical encounter plane direction, then there will likely be little encounter rate dependency as that component's threshold is increased or decreased.

7.3 Probability-based criteria

In contrast to the distance-based screening threshold *d*-squared relationship discussed above, there is no direct general relationship between collision probability and the number of maneuvers required. As seen the figure below, the P_C rate of change varies greatly depending on the specific aspect ratio, combined hardbody radius, miss distance, and accuracy represented by "log(σ)" being assessed. As one moves further away from the dilution ridge line (zero slope), the slope of the topology asymptotically approaches a squared relationship in the dilution region and exhibits a covariance scale factor in the confidence region.
Near the maximum probability ridge line, we have observed a quartic relationship, whereas on the so-called "dilution region" beyond the maximum probability ridge line, the slope asymptotically approaches

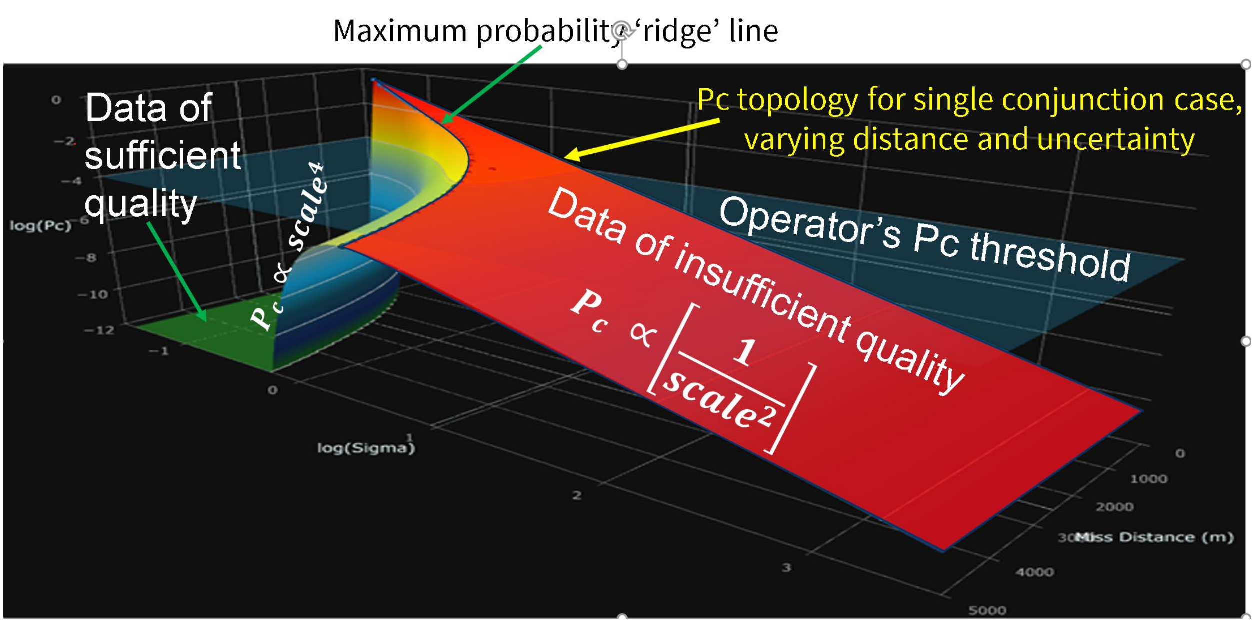

Figure 18. Example of using linked nomograms to determine P_{max} from r, d, and AR.

8. SSA DATA FUSION EXPERIMENTS

In a recent demonstration of SSA data fusion involving 14 organizations [17], it was found that comprehensive data fusion that incorporated data from government, commercial spacecraft operators, commercial SSA data and information providers and academia resulted in substantial accuracy improvements in all orbital regimes. In that demonstration, a single go/no-go criteria (collision probability) and accompanying threshold (one in ten thousand, as is commonly used by several operators particularly in LEO) was input to the processes of Section 5 to determine what positional accuracy is required to achieve such a collision probability.

Significantly, when the resultant required relative positional accuracy was allocated to both conjuncting objects equally, it was determined that the accuracy of the SSA products being used in the collision probability assessment process were generally insufficient to support the metric and corresponding threshold being operationally used to ensure flight safety. Said succinctly, the SSA data quality did not support the way it was being used. Three ways to address this troubling finding are: (1) use data fusion and invite data exchange necessary to achieve the required accuracy; (2) reconsider the go/no-go metrics being used to determine risk; and (3) reevaluate the threshold(s) being employed to ensure that existing positional data accuracies are harmonized with the thresholds and metrics the operator uses.

9. CONCLUSION

Perhaps one of the biggest shortcomings in our use of SSA data today is the general lack of effort to assure ourselves that SSA data is sufficiently accurate to support the purpose intended. In this paper, we listed many of the collision avoidance maneuver Go/No-Go criteria available to the operator. We then summarized, for twenty spacecraft operators who collectively operate almost 600 spacecraft spanning all orbit regimes, the diverse/disparate metrics and corresponding thresholds that they use operationally.

When miss distance-based screening thresholds are set to encompass the maximum errors for the two conjuncting objects, the impact of using poor-quality SSA data is that the number of encounters increases as the square of the SSA data error profile. Operators then have a very difficult time knowing which potential collisions require mitigation.

Probability-based screening thresholds demand accurate orbits accompanied by realistic covariance data. Many of the probability metrics and thresholds employed by spacecraft operators today require more accurate SSA data than the operators have available to them, greatly diminishing the value of today's conjunction assessment and collision avoidance processes.

Finally, we examined relationships to map these metrics and thresholds back to typical accuracy requirements to ensure that collision avoidance processes produce meaningful, effective results. As was demonstrated in a recent STCM data fusion campaign, such accuracy requirements are often not met by legacy flight safety systems. Collaborative sharing of authoritative data (ephemerides, maneuver plans, observations, object dimensions and mass, attitude flight rules) and large-scale data fusion offer the best opportunities to ensure that actional SSA products are generated that are of sufficient accuracy to be used by spacecraft operators for their adopted Go/No-Go criteria and thresholds.

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